Adaptive Decoupled Fuzzy Sliding-Mode Control Design

Hong-Ping Lin  Lon-Chen Hung  Hung-Yuan Chung
Department of Electrical Engineering, National Central University

Abstract

In this paper, adaptive fuzzy sliding-mode controller design approach with decoupled method is proposed. The decoupled method provides a simple way to achieve asymptotic stability for a class of fourth-order nonlinear system. Using this approach, the response of system will converge faster than that of previous reports. The simulation double-inverted pendulum system is presented to demonstrate the effectiveness and robustness of the method.

Keywords: Fuzzy control, Sliding mode control

1. Introduction

Fuzzy logic controllers (FLC) are useful control schemes for plants having difficulties in deriving mathematical models or having performance limitations with conventional linear control schemes [1]. A fuzzy logic controller is designed on the basis of human experience, which means a mathematical model is not required for controlling a system. Due to this advantage, a fuzzy logic-based control has been implemented for many industrial applications [2-3].

In recent years, there have been attempts to design the FLC based on the sliding mode control (SMC) law [4-6]. They have shown that the boundary layer can be reached in finite time and the ultimate boundedness of states is obtained asymptotically even though there exist some disturbance of dynamic uncertainties of the system. Palm showed that the analogy between a simple FLC and sliding
mode controller with a boundary layer [7]. Hwang et al. proposed a fuzzy sliding mode controller and opened a way of designing an FLC for higher order nonlinear system [8]. The sliding mode control provides a good performance in the tracking of some nonlinear systems. Nevertheless, a notorious characteristic of sliding mode control approach is the discontinuity around the switching hyperplane, that means some of the state variable are vibrant. One of the methods to cope with the problem is to utilize a feedforward compensator to offset unpredictable effect of system uncertainties.

We show that the adaptive decoupled fuzzy sliding mode control (ADFSMC) has the following advantages: (1) it can well control most of complex systems without knowing their exact mathematical models. (2) The dynamic behavior of the controlled system can be approximately dominated by a fuzzified sliding surface. (3) ADFSMC can not only increase the robustness to system uncertainties but also decrease the chattering phenomenon in the conventional sliding mode controller. (4) Increasing the speed of reaching the sliding surface due to the steepest descent rule.

Moreover, another problem of designing fuzzy controllers is applying them to higher order systems. The large majority of fuzzy controllers are limited to systems with dominantly second-order dynamics. The action of such fuzzy controllers are equivalent to that of full-state feedback controllers for second-order systems and, hence, these systems can always be stabilized. However, for a fourth-order system, such as the double-inverted pendulum system [11-18], the system may not be stabilized by using a PID controller and, therefore, using a conventional fuzzy controller will result in a large number of rules. For these systems, the instinctive sense is that some rules may be not flexibility if a stabilizing rule base is determined.

In most studies, the fuzzy controller of second-order systems is designed on a phase plane built by error $e$ and change of error $\dot{e}$ that are produced from the states $x$ and $\dot{x}$. For example, in a double-inverted pendulum system only the pole subsystem is considered ignoring the cart subsystem and it is thus impossible to achieve a good control around the set point (distance=0). In this study, a decoupled fuzzy controller design is proposed. This controller guarantees some properties, such as the robust performance and stability properties. Further, a class of fourth-order nonlinear systems is investigated. Lo and Kuo [9] proposed a method called "decoupled fuzzy sliding-mode control" (DFSMC) to cope with the above issue. However, The fuzzy rule always trial-and-error. We proposed the method to find appropriate fuzzy rules in fuzzy control implementation and online adaptive rule also has the effect of improving the stability property.

The rest of the paper is divided into five sections. In Section 2, the systems are described. In Section 3, the adaptive decouple fuzzy sliding-mode control is presented. In Section 4, the proposed controller is used to control a double-inverted pendulum system. Finally, we conclude with Section 5.

2. System description
Consider a second-order nonlinear system, which can be represented by the following state-space model in a canonical form:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f(x) + b(x)u + d(t) \\
f(t) &= x_3(t)
\end{align*}
\]  

where \( x = [x_1, x_2, x_3]^T \) is the state vector, \( f(x) \) and \( b(x) \) are nonlinear functions, \( u \) is the control input, and \( d(t) \) is external disturbance. The disturbance is assumed to be bounded as \( |d(t)| \leq D(t) \).

For this kind of the second order system, we can use many kinds of control methods, such as, fuzzy control, PID control, sliding mode control etc. A control law \( u \) can be easily designed to make the second order system (1) arrive at our control goal. However, for such nonlinear models as a double-inverted pendulum system, the system dynamic representation is generally not in a canonical form exactly. Rather, it has a form shown below:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f_1(x) + b_1(x)u_1 + d_1(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= f_2(x) + b_2(x)u_2 + d_2(t)
\end{align*}
\]  

where \( x = [x_1, x_2, x_3, x_4]^T \) is the state vector, \( f_1(x), f_2(x) \) and \( b_1(x), b_2(x) \) are nonlinear functions, \( u_1, u_2 \) are the control inputs, and \( d_1(t), d_2(t) \) are external disturbances. The disturbances are assumed to be bounded as \( |d_1(t)| \leq D_1(t), |d_2(t)| \leq D_2(t) \). From (2), one can design \( u_1 \) and \( u_2 \) respectively, however, this approach is only utilized to control a subsystem in (2). For example, if the model is a double-inverted pendulum system, we only control either the pole or the cart of a system such as (2). Hence, the idea of decoupling is employed to design a control \( u \) to govern the whole system.

The switching line is defined by:

\[
s : s c_s x = 0
\]  

For the second order system (1), a switching line is chosen as

\[
s = c_1 x_1 + x_2
\]  

By taking the time derivative of both sides of (4), we can obtain

\[
\dot{s} = c_1 \dot{x}_2 + \dot{x}_1 = c_1 x_2 + f(x) + b(x)u + d
\]  

Then, multiplying both sides of the above equation by \( s \) gives

\[
s \dot{s} = s c_1 x_2 + sf(x) + sb(x)u + sd
\]  

Here, we assume that \( b(x) > 0 \). In (5), it is seen that \( \dot{s} \) increases as \( u \) increases and vice versa. Equation (6) provides the information that if \( s > 0 \), the increasing \( u \) will make \( s \dot{s} \) decrease and that if \( s < 0 \), the increasing \( u \) will make \( s \dot{s} \) decrease.

3. Design of adaptive decoupled fuzzy logic controller
In this section, the idea of the signed distance of fuzzy logic control is used in section 3. For implementation, a triangular type membership function is chosen for the aforementioned fuzzy variables, as shown in Fig. 1. In Eqs. (2), we first define one switching line as

\[ s_1 = c_1(x_1 - z) + x_2 \]  

(7)

and another switching line as

\[ s_2 = c_2x_1 + x_4 \]  

(8)

The control objective is to drive the system state to the original equilibrium point. The switching line variables \( s_1 \) and \( s_2 \) are reduced to zeros gradually at the same time by an intermediate variable \( z \), as illustrated in Fig. 2.

In equation (7), \( z \) is a value transferred from \( s_2 \), it has a value proportional to \( s_2 \) and has the range proper to \( s_1 \). Equation (7) denotes that the control objective of \( u \) is changed from \( s_1 = 0 \), \( s_2 = 0 \) to \( s_1 = z \), \( s_2 = 0 \).

Because the controller \( u = u_1 \) is used to govern the whole system, the bound of \( x_1 \) can be guaranteed by letting

\[ |z| \leq Z_0, \quad 0 < Z_0 < 1 \]  

(9)

where \( Z_0 \) is the upper bound of \( \text{abs}(z) \). Equation (9) implies that the maximum absolute value of \( x_1 \) will be limited.

Summarizing what we have mentioned above, \( z \) can be defined as

\[ z = \text{sat}(s_1, \Phi_1) - Z_0, \quad 0 < Z_0 < 1 \]  

(10)

where \( \Phi_1 \) is the boundary layer of \( s_1 \) to smooth \( z \). \( \Phi_1 \) transfers \( s_1 \) to the proper range of \( x_1 \), and the definition of \( \text{sat}(\cdot) \) function is

\[ \text{sat}(p) = \begin{cases} \text{sgn}(p), & \text{if } |p| \geq 1 \\ \phi, & \text{if } |p| < 1 \end{cases} \]  

(11)

Notice that \( z \) is a decaying oscillation signal because \( Z_0 \) is a factor less than one.

The ADFSMC is proposed to control the whole system states to approach to zeros with satisfactory transient responses. The control objective of the ADFSMC is to let the sliding surface variables \( s_1 \) and \( s_2 \) will simultaneously converge to zeros, and then the two subsystems \((x_1, x_4)\) and \((x_2, x_3)\) will also converge to zeros simultaneously. A principal diagram for ADFSMC is given in Fig. 2, which includes two fuzzy inference systems, the slope inference rules, and the fuzzy sliding-mode control rules. The singleton fuzzy sliding-mode control rules are constructed and tuned using the idea that the state can quickly reach the hybrid sliding surface without large overshoot.

**Remark.** Consider equation (7). If \( s_1 = 0 \), then \( x_1 = z, x_2 = 0 \). Since \( z \) is a value transferred from \( s_2 \), when \( z \to 0 \), then \( x_1 \to 0 \) and \( x_2 \to 0 \). From equation (8), if the condition \( z \to 0 \), the control objective can be achieved.
Fig. 1. (a) Fuzzy input membership functions. (b) Fuzzy output membership functions. (c) Fuzzy rules of an adaptive fuzzy sliding mode controller

Fig. 2. Adaptive decoupled fuzzy sliding mode control system block diagram

As described in the last section, the objective of the SMC is to design a control law (12) so that the reaching condition (8) is satisfied. In this section, we will develop an adaptive fuzzy logic control law [13] in an attempt to accomplish this object. The proposed adaptive fuzzy control is called the adaptive fuzzy sliding mode control because it is based on the principle of SMC. The proposed ADFSMC has on line tuning fuzzy rule without the trial-and-error process for finding appropriate fuzzy rules where \( s \) and \( \dot{s} \) are the inputs and \( u \) denote the output. The sliding surface variable \( s \) is employed as the one dimensional fuzzy input variable. The control law is derived from the fuzzy inference decision and defuzzification operation.
\[
\sum_{i=1}^{n} \mu_i p_i^{m_i} \sum_{i=1}^{n} \mu_i p_i^{m_i} = \sum_{i=1}^{n} \mu_i x_i \sum_{i=1}^{n} \mu_i x_i
\]

where \( n \) is the number of rules and \( x_i \) is the consequent parameter. \( \mu_i \) is the weight for the corresponding rule which has activated. \( \varphi_i \) is the weight of each singleton fuzzy subset for constituting the control law \( u \). In the implementation, \( \dot{x} \) is approximated with \((x-S_k)T\) where \( T \) is the sampling period. For simplicity, a triangular type membership function is chosen for the aforementioned adaptive fuzzy variables. The tunable consequent parameters of those peaks of singleton membership functions can be set zero as an initial condition. A novel online parameters tuning algorithm is proposed to adjust the consequent parameters for monitoring the system control performance. The adaptive rule is derived from the steepest descent rule to decrease the value of \( \dot{x} \) with respect to \( x' \). Then, the modification equation of the consequent parameter is

\[
\dot{\lambda}_i = -\eta \frac{\partial h_i(x)}{\partial \lambda_i(x)}
\]

Based on the chain rule, the previous equation can be rewritten as

\[
\dot{\lambda}_i = -\eta \frac{\partial h_i(x)}{\partial \lambda_i(x)} = \eta h(x) \frac{\partial h_i(x)}{\partial \lambda_i(x)}
\]

\[
= \eta h(x) \sum_{i=1}^{n} \mu_i(x)
\]

where the adaptive rate parameter, \( \eta \), and the system input parameter, \( h(x) \), are combined as a learning rate parameter, \( \delta \), in order to release the requirement of using this system input parameter for practical implementation. Then, the central positions of the defuzzification membership functions can be regulated directly through the modification of consequence parameter, \( \lambda ' \). Hence, it achieves the objective of online learning and fuzzy rules adjustment. This adaptive rule has two main contributions to this proposed model-free fuzzy sliding mode control. First, it eliminates the trial-and-error process for finding appropriate fuzzy rules in fuzzy control implementation. Second, this online adaptive rule also has the effect of improving the stability property and increasing the speed of reaching the sliding surface due to the steepest descent rule.

4. Computer Simulations

In this section, we shall demonstrate that the ADFSMC is applicable to both the double-inverted pendulum system [9] to verify the theoretical development.

The structure of a double-inverted pendulum system is illustrated in Fig. 3. Pole 1 is the pole connected to the cart and pole 2 is the one above pole 1. The system’s dynamics is represented by
\[ x_1 = x_3 \]
\[ x_2 = f_1 + b_1 u + d \]
\[ x_3 = y_1 \]
\[ x_4 = f_2 + b_2 u + d \]
\[ x_5 = y_2 \]
\[ x_6 = f_3 + b_3 u \]  \hspace{1cm} (15)

where:
\( x_1 = \theta_1 \) angle of pole 1 with respect to the vertical axis;
\( x_2 = \theta_1 \) angular velocity of pole 1 with respect to the vertical axis;
\( x_3 = \theta_2 \) angle of pole 2 with respect to the vertical axis;
\( x_4 = \theta_2 \) angular velocity of pole 2 with respect to the vertical axis;
\( x_5 = x \) position of the cart;
\( x_6 = v \) velocity of the cart.

In what follows, we define the following variables:
\[ s_1 = c_1 (\theta - z) + \theta_1 c_1 (\theta - z) + x_1 \]  \hspace{1cm} (16)
\[ s_2 = c_2 x + \theta_2 c_1 x + x_1 \]  \hspace{1cm} (17)

and
\[ z = \text{sat}(s_1, \Phi_s) \cdot Z_w, \quad 0 < Z_w < 1 \]  \hspace{1cm} (18)

In the simulation, the following specifications are used:
\( l_1 = 1 \) m, \( l_2 = 1 \) m, \( m_1 = 1 \) kg, \( m_2 = 1 \) kg, \( m_3 = 1 \) kg, \( L = 0.5 \) m, \( g = 9.8 \) m/s\(^2\), \( c_1 = 5 \)
\( \Phi_1 = 15, \, \Phi_2 = 15, \, Z_w = 0.4712, \, |d| < 0.0873, \, K = 10 \).

Initial values are
\( \delta_1 = 30^\circ, \, \delta_2 = 10^\circ, \, \Phi_1 = 0, \, \Phi_2 = 0, \, x = 0, \, v = 0. \)

The simulation result is found that the pole and the cart can be stabilized to the equilibrium point.
Fig. 3. Structure of a double-inverted pendulum system

Fig. 4. Angle evolution of the pole 1.
(solid line is the ADFSINC control and dotted line is the DFSINC [9])
5. Conclusion

The adaptive decoupled fuzzy sliding mode controller has been presented. This control strategy establishes the appropriate fuzzy rules by continuous online learning instead of by a trial-and-error process. It simplifies the implementation of a fuzzy controller. In addition, only five rules are required for each dynamic operation instead of two-dimensional fuzzy rules bank. It can reduce the computing time and data base requirements. The design parameters are not sensitive to the control performance. The response of system will converge faster than that of previous reports. Next, simulation results show that the pole and the cart can be stabilized to the equilibrium.
6. References


[18] Zhang, Z., Shi, X. and Li, J., Fuzzy control of a double inverted pendulum based on T-S model.