Decoupled Control Using Neural Network-Based Sliding Mode Controller for Nonlinear Systems

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Abstract

In this paper, adaptive neural network sliding-mode controller design approach with decoupled method is proposed. The decoupled method provides a simple way to achieve asymptotic stability for a class of fourth-order nonlinear system. The adaptive neural sliding mode control system is comprised of neural network (NN) and a compensation controller. The NN is the main tracking controller, which is used to approximate an ideal computational controller. The compensation controller is designed to compensate for the difference between the ideal computational controller and the neural controller. An adaptive methodology is derived to update weight parts of the NN. Using this approach, the response of system will converge faster than that of previous reports. The simulation results for ball-beam system are presented to demonstrate the effectiveness and robustness of the method.

Keywords: Neural Network, Sliding mode, Ball-beam system

1. Introduction

Various implementation methodologies for sliding mode controllers exist today. variable structure control (VSC) with sliding mode, or sliding-mode control (SMC), is one of the effective nonlinear robust control approaches since it provides system dynamics with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode [1-4]. It possesses many advantages including: (i) insensitivity to parameter variations; (ii) external disturbance rejection; and (iii) fast dynamic responses. However, there is undesirable chattering in the control effort and bounds on the uncertainties are required in the design of the SMC. The uncertainties usually include unmodel dynamics, parameter variations and external disturbances, etc. If the actual bounds of the uncertainties exceed the assumed values designed in the controller, stability of the system in not guaranteed. Like other conventional control structures, the design of sliding mode controllers needs the knowledge of the mathematical model of the plant, which decreases the performance in some applications where the mathematical modeling of the system is very hard and where the system has a large range of parameter variation together with unexpected and sudden external disturbances. That controller should also adapt itself to large parameter variations and to unexpected external disturbances [15].

For those cases we need a controller are generally called “intelligent” controllers. These controllers mainly work on the principals of fuzzy-logic, neural networks (NN), genetic algorithms etc. The idea of combining these intelligent control structures with sliding mode approach attracted many researches [5-14].

Recently, NN-based stable and on-line adaptive control has been paid much attention in NN applications in robot control of trajectory tracking [7,13,14]. These researches have two common features: (1) locally generalizing networks are usually used for fast learning or adaptation, examples of this class of networks include the Gaussian radial basis function (RBF) like nets [17] neural networks (NN) combined with sliding mode are used to design the adaptive control architecture for the active dynamic absorber system. The cerebellar model articulation controller (CMAC) [10] used to the piezoelectric actuated tool post. The basis spline network and a certain class of fuzzy logic network [16] applied to friction compensation. (2) Lyapunov stability theory or passive theory is employed to design a closed loop control system, thus providing global stability [11,14].

In this paper, we develop a decoupled sliding mode control (DSMC) design strategy based on NN. The weights of the NN are changed according to some adaptive algorithm for the purpose of controlling the system states to hit an user-defined sliding surface and then slide along it. The initial weights of the NN can be set to small random numbers, and then on-line tuned, no supervised learning procedures are needed. This makes NN suitable for the nonlinear dynamic system control.

A decoupled neural networks-based sliding-mode control (DNNSMC) design scheme is presented. An adaptive law is employed to on-line adjust the weights of radial basis functions by using the reaching condition of a specified sliding surface. Since the proposed structure is able to learn the weights of the NN continuously, the initial weights can be started from zero for a class of fourth-order nonlinear systems. Each subsystem, which is decoupled into two second-order systems, is said to have main and sub-control purpose. Two sliding surfaces are constructed through the state variables of the
decoupled subsystem. We define main and sub-target condition for these sliding surfaces, and introduce an intermediate variable from the sub-sliding surface condition. The proposed adaptation law, which results from the direct adaptive approach, is used to appropriately determine the width of the unknown system variables.

The on-line adjust algorithm is derived in the Lyapunov sense; thus, the stability of the control system can be guaranteed. Furthermore, to relax the requirement for the uncertain bound in the compensation controller, an estimation mechanism is investigated to observe the uncertain bound, so that the chattering phenomena of the control efforts can be relaxed. To illustrate the effectiveness of the proposed design method, a comparison between a DFSMC [9] and the proposed DNNSMC is made.

We proposed the DNNSMC has the following advantages: (1) It can well control most of complex systems without knowing their exact mathematical models. (2) The dynamic behavior of the controlled system can be approximately dominated by a hybrid sliding surface. (3) DNNSMC can not only increase the robustness to system uncertainties but also decrease the chattering phenomenon in the conventional sliding mode controller. (4) Our control approach has the advantage over the model-based control scheme to the former that it does require a prior knowledge of dynamic nonlinear system.

The rest of the paper is divided into five sections. In Section 2, review the sliding mode control. In Section 3, design decoupled neural network sliding mode controller is described. In Section 4, the proposed controller is used to control a ball-beam system. Finally, we conclude with Section 5.

2. System description

Consider a second-order nonlinear system, which can be represented by the following state-space model in a canonical form:

\[ \begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f(x) + b(x)u + d(t) \\
y(t) &= x_1(t)
\end{align*} \]

where \( x = [x_1, x_2]^T \) is the state vector, \( f(x) \) and \( b(x) \) are nonlinear functions, \( u \) is the control input, and \( d(t) \) is external disturbance. The disturbance is assumed to be bounded as \( |d(t)| \leq D(t) \).

For this kind of the second order system, we can use many kinds of control methods, such as, fuzzy control, PID control, sliding mode control...etc. A control law \( u \) can be easily designed to make the second order system (1) arrive at our control goal. However, for such nonlinear models as a cart-pole system, the system dynamic representation is generally not in a canonical form exactly. Rather, it has a form shown below

\[ \begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f_1(x) + b_1(x)u_1 + d_1(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= f_2(x) + b_2(x)u_2 + d_2(t)
\end{align*} \]  \tag{2}

where \( x = [x_1, x_2, x_3, x_4]^T \) is the state vector, \( f(x) \), \( f_1(x) \) and \( b(x) \), \( b_1(x) \) are nonlinear functions, \( u_1 \), \( u_2 \) are the control inputs, and \( d_1(t) \), \( d_2(t) \) are external disturbances. The disturbances are assumed to be bounded as \( |d_1(t)| \leq D_1(t) \), \( |d_2(t)| \leq D_2(t) \). From (2), one can design \( u_1 \) and \( u_2 \) respectively, however, this approach is only utilized to control a subsystem in (2). For example, if the model is a cart-pole system, we only control either the pole or the cart of a system such as (2). Hence, the idea of decoupling is employed to design a control \( u \) to govern the whole system.

In Eqn. (2), we first define one switching line as

\[ \begin{align*}
s_1 &= c_1(x_1 - z) = [c_1, 1][x_1, x_2]^T - c_2z \\
&= c^Tx_2 - c_2z \tag{3}
\end{align*} \]

and another switching line as

\[ s_2 = c_2x_3 + x_4 \tag{4} \]

The control objective is to drive the system state to the original equilibrium point. The switching line variables \( s_1 \) and \( s_2 \) are reduced to zeros gradually at the same time by an intermediate variable \( z \). In equation (3), \( z \) is a value transferred from \( s_2 \), it has a value proportional to \( s_1 \) and has the range proper to \( x \). Equation (3) denotes that the control objective of \( u_1 \) is changed from \( x_1 = 0 \), \( x_2 = 0 \) to \( x_1 = z \), \( x_2 = 0 \).

Because the controller \( u = u_1 \) is used to govern the whole system, the bound of \( x_1 \) can be guaranteed by letting

\[ |z| \leq Z_{upper}, \quad 0 < Z_{upper} < 1 \tag{5} \]

where \( Z_{upper} \) is the upper bound of \( abs(z) \). Equation (5) implies that the maximum absolute value of \( x_1 \) will be limited.

Summarizing what we have mentioned above, \( z \) can be defined as

\[ z = sat(s_2 / \Phi_z) \cdot Z_{upper}, \quad 0 < Z_{upper} < 1 \tag{6} \]

where \( \Phi_z \) is the boundary layer of \( s_2 \) to smooth \( z \), \( \Phi_z \) transfers \( s_2 \) to the proper range of \( x_1 \), and the definition of \( sat(\cdot) \) function is

\[ sat(\phi) = \begin{cases} 
\text{sgn}(\phi), & \text{if } |\phi| \geq 1 \\
\phi, & \text{if } |\phi| < 1
\end{cases} \tag{7} \]

Notice that \( z \) is a decaying oscillation signal because \( Z_{upper} \) is a decaying oscillation signal because
Remark 1. Consider equation (3). If \( s_i = 0 \), then \( x_i = z \), \( x_2 = 0 \). Since \( z \) is a value transferred from \( s_2 \), when \( s_2 \rightarrow 0 \), then \( z \rightarrow 0 \) and \( x_2 \rightarrow 0 \). From equation (4), if the condition \( s_i \rightarrow 0 \), the control objective can be achieved. Moreover, the choice of \( c_1 \) and \( c_2 \) has strong influence on the behavior in the transient state of the system. Appropriate choice of and is necessary for achieving favorable transient response.

In the design of decoupled sliding mode controller, an equivalent control is first given so that the states can stay on sliding surface. Thus, in sliding motion, the system dynamic is independent of the original system and a stable equivalent control system is achieved. The equivalent control can be obtained by letting \( s_1 \) equal to zero. That is

\[
\dot{s}_1 = c_1(x_1-z) + x_1 = c_1x_1 - c_1z + f_1 + h_1u + d_1 = 0 \quad (8)
\]

\[
u_{eq} = \frac{1}{b_1}(x(t) - f(t) - d(t)) + s_1(t) + \lambda s_1(t)) \quad (9)
\]

where \( s_1 = s_1 - \Phi_1 \cdot \text{sat}(s_1/\Phi_1) \). Substituting Eq. (9) into Eq. (8), we obtain

\[
\dot{s}_1(t) + \lambda s_1(t) = 0 \quad (10)
\]

Since \( \lambda \) is a positive value, the sliding surface on the phase plane can be defined as (3). The sliding surface on the phase plane can be defined as (3). This sliding variable, \( s_1 \), will be used as the input signal for establishing the neural network to calculate the control law. The presentation of DNNSMC will be discussed in the Section 3.

3. Design of Decoupled Neural Network Sliding Mode Controller

In this section, we show how to develop a decoupled neural network sliding mode controller for obtaining the equivalent control through weight adaptation. Then, we construct the hitting control to guarantee system’s stability. In this paper, the NN is used as such the nonlinear approximator. If the state trajectory can be forced to slide on sliding surface, then a stable equivalent control system is achieved. However, if the functions \( f \) and \( b \) are unknown, there is no way to yield equivalent control \( u_{eq} \). In this paper, a set of neural network base is applied to approximating (9). Motivated by the principle of sliding mode control (SMC), the control law consists of the following two parts, one is the estimated sliding component \( u_{DNNSM} \) that constructed by an adaptive mechanism. The effect of this term is to force the system state to slide on the sliding surface. Another is the hitting control \( u_h \) that drives the states toward the sliding surface. Thus the control law can be represented as

\[
u = u_{DNNSM} + u_h \quad (11)
\]

where \( u_{DNNSM} \) is approximate equivalent control and \( u_h \) is hitting control.

3.1 Basic idea of NN approximation

Here, we employ a simple two-layer NN to approximate a general smooth nonlinear function on a compact set \( S \in \mathbb{R}^n \). According to the NN approximation property, we have

\[
f(x) = \mathbf{w}^T \sigma(\mathbf{v}^T x) + e(x) \quad (12)
\]

where \( x = [x_1 \cdots x_p]^T \) is the input to NN, \( \sigma(\cdot) \) is an active function, where \( \mathbf{W} = [w_1 w_2 \cdots w_m]^T \) and \( \mathbf{V} = [v_1 v_2 \cdots v_n]^T \) defined as the collection of NN weights for output and hidden layer, respectively, and \( e(x) \) is the NN approximation error. Hence, tuning of NN weights also involves tuning of thresholds as well. For convenience of narration, let us define NN weight error \( \mathbf{w} = \mathbf{w} - \mathbf{w}^* \), \( \mathbf{v} = \mathbf{v} - \mathbf{v}^* \) (\( "^*" \) represents estimation value), denote the norm of a vector \( x \) is defined by \( ||x|| = \sqrt{x^T x} \), and the norm of a vector function \( f(x) \) is \( ||f(x)|| = \sup \{|f(x)|: x \in \mathbb{R}^{p+1}\} \).

Assuming that \( f(x) \) is absolutely integrable, i.e.

\[
\int_{-\infty}^{\infty} |f(x)| \, dx \leq k \quad (13)
\]

where \( k \) is a sufficiently large positive constant. We can establish the following approximation theorem.

**Theorem 1:** For every function \( f(x) \) satisfying (13) and every sigmoidal function \( \sigma(x) = 1/(1+exp(-x)) \), if \( \xi \geq \sqrt{n} \ln n \) (\( n \) is the number of the hidden-layer neurons), there exists an NN functional estimate \( \hat{f}_e(x) = W^T \sigma(V^T x) \in G_{n,\xi} \) such that

\[
||e(x)|| \leq O(n^{1/2}) + ||f(0)|| \quad (14)
\]

where \( G_{n,\xi} = \{\sigma(\zeta(ax + b)) : \gamma \leq 2c, |a| = 1, |b| \leq 1, |a| = \sup_{x \in \mathbb{R}} |a \cdot x| \}, B \) is a bounded set in \( \mathbb{R}^{p+1} \), \( O(n^{1/2}) \) denotes the order \( n^{1/2} \) approximation.

**Proof:** Consider (12) and let

\[
\hat{f}(x) = f(x) - f(0) \quad (15)
\]

the proof of Theorem 1 can be completed based on Theorem 2 in [1].

**Remark 1:** Theorem 1 indicates that, for a special two-layer NN, the upper bound of NN functional reconstruction error is affected by NN structure (i.e. the number of hidden-layer neurons) and also related to the initial value of the estimated function.
3.2 Neural network base sliding mode control

A single-hidden-layer NN with two layers of adjustable weights is shown in Fig. 1. Following the notation used in [14], the output of this NN takes the form

$$y = \sum_{i=1}^{m} w_i \sigma(v_i s_i)$$

(16)

where \(v_i\) and \(w_i\) are the input and output of the NN, respectively; \(\sigma(\cdot)\) represents the hidden-layer activation function; \(v_i\) are the interconnection weights between the input and hidden layers; and \(w_i\) are the interconnection weights between the hidden and output layers. This architecture has one input, hidden-layer neurons, and one output. The activation function is considered as a sigmoid function

$$\sigma(s_i) = \frac{1}{1 + e^{-s_i}}$$

(17)

By collecting all the weights of the NN, (15) can be expressed in a vector form as

$$y = w^T \sigma(v s)$$

(18)

![Fig. 1. A single-hidden-layer NN with two layers.](image)

A main property of a NN regarding feedback control purpose is the universal function approximation property. A NN is capable of approximating any smooth function to any desired accuracy, provided the number of hidden-layer neurons is sufficiently large.

By the universal approximation theorem [18], there exist ideal weight vectors \(\hat{w}\) and \(\hat{v}\) such that

$$\Omega = y'(s, \hat{w}, \hat{v}) + \Delta = w^T \sigma(v s) + \Delta$$

(19)

where \(\Delta\) is the approximation error, which generally decreases as the net size increases. For any choice of a positive number \(\Delta_p\), one can find a feedforward NN such that \(|\Delta| \leq \Delta_p\) for all \(s_i\). The ideal NN weights in vectors and that are needed to best approximate a given nonlinear function are difficult to determine. In fact, they may not even be unique. However, all one needs to know for control purposes is that, for a specified value of \(\Delta_p\) some ideal approximating NN weights exists. Then, an estimate of \(\hat{\Xi}\) can be given by

$$\hat{\Xi} = \hat{y}(s, \hat{w}, \hat{v}) = \hat{w}^T \sigma(s)$$

(20)

where \(\hat{w}\) and \(\hat{v}\) are the estimated values of the ideal NN weights \(\hat{w}\) and \(\hat{v}\) that are provided by some online weight tuning algorithms subsequently to be detailed.

For notational convenience, \(\sigma(\hat{v}s_i)=[\sigma_1^\top \sigma_2^\top \cdots \sigma_n^\top]^\top\) are denoted with

$$\hat{\sigma} = \frac{1}{(1 + e^{-\chi})} = \frac{1}{(1 + e^{-x})}$$

(21)

The estimation errors of the weights of NN are defined as

$$\hat{w} = \hat{w} - w, \quad \hat{v} = \hat{v} - v$$

(22)

and the hidden-layer output error is given as

$$\hat{\sigma} = \sigma(\hat{v}s_i) - \sigma(v s_i)$$

(23)

From the function \(\sigma(x)\) with parameter \(x\), one may write its Taylor series with another parameter

$$\sigma(x') = \sigma(x) + \sigma'(x)x + O(x^2)$$

(24)

where is the Jacobian, and the last term indicates terms of order \(x^2\). Therefore,

$$\hat{\sigma} = \sigma'(v s_i)\hat{v}s_i + O(\hat{v}s_i)$$

(25)

The control law for the DNNSMC system is assumed to make the following form:

$$u_{DNNSMC}(s, \hat{w}, \hat{v}) = u_{DNNSMC}(s, \hat{w}, \hat{v}) + u_{DNNSMC}(s, \hat{w}, \hat{v})$$

(26)

where \(u_{DNNSMC}\) is the main tracking control, and the hitting control \(u_h\) is designed to stabilized the states of the control system around a pre-selected uncertainty bound.

Substituting Eq. (26) into Eq. (1), we can obtain

$$\dot{x}_2 = f_1(x) + b_1(x)u$$

$$= f_1(x) + b_1(x)(u_{DNNSMC} + u_h - u_{eq})$$

$$= c_1\dot{x}_2 + c_2 z + b_1(x)(u_{DNNSMC} + u_h - u_{eq})$$

(27)

or, equivalently

$$\dot{x}_2 = A_2x_2 + b(x)(u_{DNNSMC} + u_h - u_{eq}) + g z + \gamma s_i$$

(28)

where \(x_2 = [x_2 x_2]^\top\), \(A_2 = [0 1]^\top\), \(b(x) = [0 b_1(x)]^\top\), \(g = [0 c_2]^\top\), \(\gamma = [0 -\lambda]^\top\).

then the (8) can be rewritten
\[ \dot{s}_i = c^T A_i x_{12} - c_i z + c^T A_i x_{12} + c^T b_i (u_{\text{DNNSMC}} + u_b - u_{\text{eq}}) + c^T \gamma s_i + c^T g z - c_i z = b_i (u_{\text{DNNSMC}} + u_b - u_{\text{eq}}) - \lambda s_i \]  
\text{(29)}

where \( c = [c_i \ 1]^T \), \( s_i = s_i - \Phi_1 \cdot \text{sat}(s_i / \Phi_1) \).

Assumption 1: The following equality holds
\[ |u_{\text{DNNSMC}} - u_{\text{eq}} + \frac{1}{2} s_i \frac{\partial h_{\text{eq}}}{\partial \mathbf{x}} \mathbf{x} + \varepsilon| = E^* \]  
\text{(30)}

where the uncertainty bound \( E^* \) is a positive constant. This uncertainty bound cannot be measured for practical applications. Therefore, a bound estimation is developed to observe the bound of approximation error.

\[ \Theta = E(t) - E^* \]  
\text{(31)}

where \( E(t) \) is the estimated uncertainty bound.

The adaptive laws will be developed to adjust the parameters \( \mathbf{w}, \mathbf{v} \) and \( E \) to estimate \( \mathbf{w}^*, \mathbf{v}^* \) and \( E^* \), respectively.

Theorem 2: Consider the dynamic system described by (1) and the sliding surface (3), for the bounded, continuous desired state trajectory with bounded velocity, controller (25) can guarantee the asymptotic stability of the close-loop system. And the NN adaptive laws are given by

\begin{align*}

\mathbf{w} & = \Psi = -\gamma_1 s_i \cdot |b_i| \cdot \sigma(\mathbf{v}s_a) \\

\mathbf{v} & = \Xi = -\gamma_2 s_i \cdot |b_i| \cdot \sigma(\mathbf{v}s_a) \\

u_b & = -E \cdot |b_i| \cdot \text{sat}(s_i / \Phi_1) \\

E & = \Theta = -\gamma_3 \cdot |s_i(t)|
\end{align*}
\text{(32)-(35)}

where \( \gamma_1, \gamma_2, \) and \( \gamma_3 \) are positive constants. Moreover, the system states converge to the sliding surface asymptotically.

Proof: Choose the Lyapunov function as
\[ V = \frac{1}{2} |b_i|^2 \cdot s_i^2 + \frac{1}{2} \gamma_1 \mathbf{v}^T \mathbf{v} + \frac{1}{2} \gamma_2 \mathbf{w}^T \mathbf{w} + \frac{1}{2} \gamma_3 \Theta^2 \]  
\text{(36)}

where \( \mathbf{w} = \mathbf{w} - \mathbf{w}^*, \ \mathbf{v} = \mathbf{v} - \mathbf{v}^*, \ \Theta = E(t) - E^* \), \( \Phi_1 \)

is the boundary layer thickness, \( \mathbf{w} \) and \( \mathbf{v} \) are the vector of neural network weights, \( \mathbf{w}^* \) and \( \mathbf{v}^* \) are the optimal central position vector of the dynamic nonlinear system, and \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are positive constant. The variation of this function (36) with respect to time is

\[ \dot{V} = s_i \cdot \dot{s}_i + \frac{1}{2} s_i \cdot \frac{\partial h_{\text{eq}}}{\partial \mathbf{x}} \cdot \dot{\mathbf{x}} + \frac{1}{2} \gamma_1 \mathbf{v}^T \cdot \dot{\mathbf{v}} + \frac{1}{2} \gamma_2 \mathbf{w}^T \cdot \dot{\mathbf{w}} + \frac{1}{2} \gamma_3 \Theta^2 \cdot \dot{\Theta} \]  
\text{(37)}

By selecting appropriate values for \( \Phi_1, \) (32), (33) and (35) implies \( \dot{V} \) is negative semidefinite.
\[ \dot{V} \leq -\lambda \frac{\dot{s}^2}{|\dot{s}|} \] \quad (38)

If \(|s_\lambda| \leq \Phi_1\), \(s_\lambda = 0\). Then \(V = 0\), and \(\dot{V} = 0\).
If \(|s_\lambda| > \Phi_1\), \(s_\lambda = s_\lambda - \Phi_1 \cdot \text{sat}(s_\lambda / \Phi_1)\), and \(\dot{s}_\lambda = s_\lambda\) has the same sign as \(s_\lambda\). From the algorithm, we have \(s_\lambda = 0\). Therefore \(\dot{V} = s_\lambda \cdot s_\lambda < 0\). Then for all \(t \geq 0\), \(\dot{V} \leq 0\) holds. So is a monotonous nonincrease function. Because \(\dot{V} \geq 0\), \(\lim_{t \to \infty} V\) exists, i.e., \(V(\infty)\) exists. Then \(s_\lambda\) is bounded and \(\dot{w}\) and \(\dot{v}\) are bounded too. Since continuous function is bounded in the closure set, so \(x_1\) is bounded, and \(\dot{s}_\lambda\) is bounded too, therefore \(s_\lambda\) is uniform continuous, then \(\dot{V} = s_\lambda \cdot \dot{s}_\lambda\) is uniform continuous. Since \(V(t)\) is bounded and \(\lim_{t \to \infty} \int_{t=0}^{t} \dot{V} dt = V(\infty) - V(0)\) exists, then by Barbalat lemma [23], we have \(\lim_{t \to \infty} \dot{V} = 0\), and obtain \(\lim_{t \to \infty} s_\lambda = 0\).

In summary, the DNNSMC control law is presented in Eq. (25) with the parameters vector \(w\) and \(v\) adjusted by (32) and (33). The objective is to construct an adaptive control scheme for unknown time-dependent nonlinear plants without using a model of the plant. The proposed approach is NN based with adaptive law combine decoupled sliding mode control. Here, no prior knowledge of the plant is assumed, and the controller has to begin with exploration of the state space. The DNNSMC controller ensures Lyapunov stability of the dynamic nonlinear system.

4. Computer Simulation Results

In this section, Consider a ball-beam system as depicted in Fig. 1 and its dynamic is described below [9]:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u + d \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= B(x_3 x_2^2 - G \sin x_1) \\
\end{align*}
\]

where

\(x_1 = \theta\) the angle of the pole with respect to the vertical axis;
\(x_2 = \dot{\theta}\) the angle velocity of the pole with respect to the vertical axis;
\(x_3 = r\) the position of the cart;
\(x_4 = \dot{r}\) the velocity of the cart;

\[
B = \frac{MR^2}{J_b + MR^2} = \frac{J_b}{J_b + MR^2}; \quad J_b\text{ moment of inertia of the ball};
\]

\(M\) mass of the ball; \(R\) radius of the ball;
\(g\) acceleration of gravity.

The center of rotation is assumed to be frictionless and ball is free to roll along the beam. It is required that the ball remains in contact with the beam and that rolling occurs without slipping. The objective is to keep the ball close to the center of the beam close to the horizontal position.

In the simulation, the following specifications are used:

\[
B = 0.7143, \; J_b = 2 \times 10^{-6}, \; M = 0.05 \text{kg}, \; R = 0.01 \text{m}, \; g = 9.8 \text{m/s}^2, \; |d| \leq 0.08, c_1 = 5, c_2 = 0.5, \Phi_1 = 5, \; \gamma_1 = 0.1, \; \gamma_2 = 0.1, \; \gamma_3 = 0.1, \; Z_\omega = 0.9425, \; \Phi_1 = 0.05.
\]

initial values are

\[
\begin{align*}
x_1 &= \theta = 60^\circ, \; x_2 &= \dot{\theta} = 0, \; x_3 = 10, \; x_4 = \dot{r} = 0.
\end{align*}
\]

Fig. 2 through Fig. 4 shows the simulation result. It is found that the ball-beam system can be stabilized to the equilibrium point, and shown that \(\theta\) and \(r\) converge to zero, respectively. Further, the proposed control performance and robustness better [9].

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In the simulation, the following specifications are used:

\[
B = 0.7143, \; J_b = 2 \times 10^{-6}, \; M = 0.05 \text{kg}, \; R = 0.01 \text{m}, \; g = 9.8 \text{m/s}^2, \; |d| \leq 0.08, c_1 = 5, c_2 = 0.5, \Phi_1 = 5, \; \gamma_1 = 0.1, \; \gamma_2 = 0.1, \; \gamma_3 = 0.1, \; Z_\omega = 0.9425, \; \Phi_1 = 0.05.
\]

initial values are

\[
\begin{align*}
x_1 &= \theta = 60^\circ, \; x_2 &= \dot{\theta} = 0, \; x_3 = 10, \; x_4 = \dot{r} = 0.
\end{align*}
\]

Fig. 2 through Fig. 4 shows the simulation result. It is found that the ball-beam system can be stabilized to the equilibrium point, and shown that \(\theta\) and \(r\) converge to zero, respectively. Further, the proposed control performance and robustness better [9].
Fig. 3. System intermediate variable z of the ball-beam.

Fig. 4. Angle evolution of the beam.

5. Conclusions

In this paper, the decoupled adaptive neural sliding mode controller has been proposed in this paper. Simulation results were presented. The adaptive sliding-mode control using a dynamic network, possessing a learning law with correction-term and dead-zone, and having a time varying switching gain and boundary layer for an unknown nonlinear system is constructed. The paper investigates the application of inversion of a neural network to nonlinear control problems for which the structure of the nonlinearity is unknown. Lyapunov stability theory is used to prove the uniform ultimate boundedness of the tracking error, and simulation results demonstrate the applicability of the proposed method to achieve desired.

Reference


