Decoupling Adaptive Fuzzy Sliding-Mode Control with Rule Reduction for Nonlinear System

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Abstract—In this paper, adaptive fuzzy sliding-mode controller design approach with decoupling method is proposed. The decoupling method provides a simple way to achieve asymptotic stability for a class of fourth-order nonlinear system. The adaptive fuzzy sliding-mode control system is comprised of fuzzy controller and a compensation controller. The compensation controller is designed to compensate for the difference between the ideal computational controller and the fuzzy controller. Using this approach, the response of system will converge faster than that of previous reports. The simulation results for a ball-beam system presented to demonstrate the effectiveness and robustness of the method.

Keywords—fuzzy, mobile, robot

I. INTRODUCTION

Fuzzy logic control, as one of the most useful approaches for utilizing expert knowledge, has had extensive research in the past decade [3]. Fuzzy logic control is generally applicable to plants that are mathematically poorly modeled and where experienced operators are available for providing qualitative guiding. Although achieving much practical success, fuzzy control has not been viewed as a rigorous science, due to a lack of formal synthesis techniques which guarantee the very basic requirements of global stability and acceptable performance [13]. In stability analysis [5,16], it is commonly assumed that the mathematical model of the plants are known, this assumption contradicts the very basic premise of fuzzy control systems, i.e., to control processes that are poorly modeled from a mathematical view. Based on fuzzy systems which are capable of approximating, with arbitrary accuracy, any real continuous function on a compact set, a globally stable adaptive controller is firstly synthesized from a collection of fuzzy IF-THEN rules [2]. The fuzzy system, used to approximate an optimal controller, is adjusted by an adaptive law based on a Lyapunov function synthesis approach.

In recent years, there have been attempts to design the fuzzy based on the sliding-mode control law [1,4,6-12,14,15]. They have shown that the boundary layer can reached in finite time and the ultimate boundedness of states is obtained asymptotically even though there exist some disturbance of dynamic uncertainties of the system. Palm showed that the analogy between a simple and sliding-mode controller with a boundary layer [14]. Hwang et al. proposed a fuzzy sliding-mode controller and opened a way of designing for higher order nonlinear system [8]. The sliding-mode control provides a good performance in tracking of some nonlinear systems. Nevertheless, a notorious characteristic of sliding-mode control approach is the discontinuity around the switching hyper-plane, that means some of the state variable are vibrant. One of the methods to cope with the problem is to utilize a feed-forward compensator to offset unpredictable affect of system uncertainties.

In most studies, the fuzzy controller of second-order systems is designed on a phase plane built by error $e$ and change of error $\dot{e}$ that are produced from the states $x$ and $\dot{x}$. For example, in a cart-pole system only the pole subsystem is considered ignoring the cart subsystem and it is thus impossible to achieve a good control around the set point (distance=0). In this study, a decoupling fuzzy controller design is proposed. This controller guarantees some properties, such as the robust performance and stability properties. Further, a class of fourth-order nonlinear systems is investigated.

A decoupling adaptive fuzzy sliding-mode control design scheme is presented through width of consequence adaptation for a class of fourth-order nonlinear systems. Each subsystem, which is decoupling into two second-order systems, is said to have main and sub-control purpose. Two sliding surfaces are constructed through the state variables of the decoupling subsystem. We define main and sub-target condition for these sliding surfaces and introduce an intermediate variable from the sub-sliding surface condition. The proposed adaptation law, which results from the direct adaptive approach, is used to appropriately determine the width of the unknown system variables. And the membership functions in the THEN part will vary with the width adaptation of consequence that one tuning factor is characterized to adapt the control rules.

A tuning methodology is derived via the rules regulation. The online tuning algorithm is derived in the Lyapunov sense; thus, the stability of the control system can be guaranteed. To illustrate the effectiveness of the proposed design method, a comparison between a decoupled fuzzy sliding-mode control [12] and the proposed control is made.

The rest of the paper is divided into five sections. In Section 2, the systems are described. In Section 3, the adaptive
decoupling fuzzy sliding-mode control is presented. In Section 4, the proposed controller is used to control a ball-beam system. Finally, we conclude with Section 5.

II. SYSTEM DESCRIPTION

Consider a four-order nonlinear system, which can be represented by the following state-space model in a canonical form:

\[ \begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f_1(x) + b_1(x)u_1 + d_1(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= f_2(x) + b_2(x)u_2 + d_2(t)
\end{align*} \tag{1} \]

where \( x = [x_1 \ x_2 \ x_3 \ x_4]^T \) is the state vector, \( f_1(x), f_2(x) \) and \( b_1(x), b_2(x) \) are nonlinear functions, \( u_1, u_2 \) are the control inputs, and \( d_1(t), d_2(t) \) are external disturbances. The disturbances are assumed to be bounded as \( |d_1(t)| \leq D_1(t), \ |d_2(t)| \leq D_2(t) \). From (1), one can design \( u_1 \) and \( u_2 \), respectively, however, this approach is only utilized to control a subsystem in (1). For example, if the model is a cart-pole system, we can only control either the pole or the cart of a system such as (1). Hence, the idea of decoupling is employed to design a control \( u \) to govern the whole system.

In Eqn. (1), we first define one switching line as

\[ s_1 = c_1(x_1 - z) + x_2 \]

\[ = [c_1 \ I][x_1 \ x_2]^T - c_2z \tag{2} \]

and another switching line as

\[ s_2 = c_2x_2 + x_4 \tag{3} \]

The control objective is to drive the state to the original equilibrium point. The switching line variables \( s_1 \) and \( s_2 \) are reduced to zeros gradually by an intermediate variable \( z \). In equation (3), \( z \) is a value transferred from \( s_2 \), it has a value proportional to \( x_2 \) and has the range proper to \( x_1 \). Equation (3) denotes that the control objective of \( u_1 \) is changed from \( x_1 = 0, x_2 = 0 \) to \( x_1 = z, x_2 = 0 \).

Because the controller \( u = u_1 \) is used to govern the whole system, the bound of \( x_1 \) can be guaranteed by letting

\[ |z| \leq Z_{upper}, \quad 0 < Z_{upper} < 1 \tag{4} \]

where \( Z_{upper} \) is the upper bound of \( abs(z) \). Equation (4) implies that the maximum absolute value of \( x_1 \) will be limited. Summarizing what we have mentioned above, \( z \) can be defined as

\[ z = sat(s_2, \Phi_z) \cdot Z_{upper}, \quad 0 < Z_{upper} < 1 \tag{5} \]

where \( \Phi_z \) is the boundary layer of \( s_2 \) to smooth \( z \), \( \Phi_z \) transfers \( s_2 \) to the proper range of \( x_1 \), and the definition of \( sat(\cdot) \) function is

\[ sat(\varphi) = \begin{cases} 
\text{sgn}(\varphi), & \text{if } |\varphi| \geq 1 \\
\varphi, & \text{if } |\varphi| < 1 
\end{cases} \tag{6} \]

Notice that \( z \) is a decaying oscillation signal because \( Z_{upper} \) is a factor less than one.

Remark 1. Consider equation (3). If \( x_1 = 0 \), then \( x_2 = z \), \( x_3 = 0 \). Since \( z \) is a value transferred from \( s_2 \), when \( s_2 \to 0 \), then \( z \to 0 \) and \( x_1 \to 0 \). From equation (4), if the condition \( s_1 \to 0 \), the control objective can be achieved. Moreover, the choice of \( c_1 \) and \( c_2 \) has strong influence on the behavior in the transient state of the system. Appropriate choice of \( c_1 \) and \( c_2 \) is necessary for achieving favorable transient response.

In the design of the sliding-mode controller, an equivalent control is first given so that the states can stay on sliding surface. Thus, in sliding motion, the system dynamic is independent of the original system and a stable equivalent control system is achieved. The equivalent control can be obtained by letting \( s_1 \) equal to zero. That is

\[ s_1 = c_1(x_1 - z) + x_2 = c_1x_1 - c_2z + f_1 + b_1u + d_1 = 0 \tag{7} \]

\[ u_{eq} = \frac{1}{b_1}(-c_1x_1 + c_2x_2 - f_1 + s_i + \lambda s_i) \tag{8} \]

Substituting Eq. (8) into Eq. (7), we obtain

\[ s_i + \lambda s_i = 0 \tag{9} \]

Since \( \lambda \) is a positive value, the sliding surface on the phase plane can be defined as (3). The sliding surface on the phase plane can be defined as (3).

This sliding variable, \( s_1 \), will be used as the input signal for establishing a fuzzy rule to calculate the control law. The presentation of proposed control will be discussed in the Section 3. formatter will need to create these components, incorporating the applicable criteria that follow.

III. DESIGN OF THE PROPOSED CONTROL

In this section, we show how to develop a fuzzy sliding-mode control with rule reduction for obtaining the equivalent control through rule adaptation in decoupling system. Then, we construct the correct control to guarantee system’s stability. The proposed FSMC has on-line self-tuning fuzzy rule without the trial-and-error process for find appropriate fuzzy rule. If the state trajectory can be forced to slide on sliding surface, then a stable equivalent control system is achieved. However, if the function \( f_i \) is unknown, there is no way to yield equivalent control \( u_{eq} \). In this paper, a set of fuzzy rule base is applied to approximating (8). Motivated by the principle of decoupled sliding-mode control, the control law consists of the following two parts, one is the estimated decoupled sliding component \( u_i \) that constructed by an adaptive mechanism, as shown in Fig. 1. The effect of this term is to force the system state to slide on the sliding surface. Another is the correct control \( u_1 \) that drives the states toward the sliding surface. Thus the control law can be represented as
\[ u = u_f + u_c \] 

where \( u_f \) is approximate equivalent control and \( u_c \) is correct control.

Now, the rule base of FSMC is constructed as (\( i,j \))th rule: If \( s_i \) is \( A_i \) and \( s_j \) is \( B_j \) then \( u_f \) is \( u_{ij} \);

where

\[
i \in I = \{-n, -n + 1, \ldots, -1, 0, 1, \ldots, n-1, n\} \tag{11}\]

\[
j \in J = \{-m, -n + 1, \ldots, -1, 0, 1, \ldots, m-1, m\} \tag{12}\]

\[
k \in K = \{-l, -l + 1, \ldots, -1, 0, 1, \ldots, l-1, l\} \tag{13}\]

Now, the rule base of FSMC is constructed as in this study, the triangular-typed and singletons are, respectively, used to define the membership function of IF-part and THEN-part, which are shown in Figs. 2-4. The membership functions of IF-part and THEN-part are arranged as having the same width and boundary in the universe of discourse \([-y; p] \times [-q; q]\) and \([-U; U]\), respectively. The rule is defined in the following analytic form [10]:

\[
k = -(\alpha i - (1-\alpha) j), \quad \alpha \in [0,1] \tag{14}\]

where \( x \) is an operation that takes an integer which is the nearest to \( x \) and \( \alpha \) is a rule regulating factor. Obviously, by properly adjusting \( \alpha \), the value of \( k \) will be changed by (14), that indirectly determine which \( u_c \) should be taken into account. So the controller is expected to provide different control actions corresponding to different \( \alpha \). According to (11)-(13), it is easily found that (14) is constrained by

\[-l \leq \alpha i + (1-\alpha) j \leq l \tag{15}\]

Considering the extreme case, \( i=n \) and \( j=m \), the following condition shows that \( \alpha \) should be satisfied:

\[
\alpha \leq \frac{l-m}{n-m} \tag{16}
\]

It also shows that \( l = n > m \) to satisfy the condition \( \alpha \in [0,1] \).

Define each membership function of THEN-part as

\[
u_k = k_i h \tag{17}\]

where \( h = U/l \), \( k_i \) is the modified function of \( k \) and is represented as

\[
k_i = -(\alpha i - (1-\alpha) j) \tag{18}\]

Without loss of generality, consider the case of \( p_i \leq s_i \leq p_{i+1} \), and \( q_i \leq s_i \leq q_{i+1} \), thus there are four rules are fired

\((i;j); (i;j+1); (i+1;j); (i+1;j+1)\). \tag{19}\]

Therefore, we can get the crisp output through defuzzification strategy

\[
u_f = \frac{N}{M} \tag{20}\]

where

\[
N = w_i u_i + w_2 u_2 + w_3 u_3 + w_4 u_4
\]

\[
w_i = \min\{\mu_a(s_i), \mu_b(s_j)\}
\]

\[
w_2 = \min\{\mu_{a,s}(s_i), \mu_{b,s}(s_j)\}
\]

\[
w_3 = \min\{\mu_{a,s}(s_i), \mu_{b,s}(s_j)\}
\]

\[
w_4 = \min\{\mu_{a,s}(s_i), \mu_{b,s}(s_j)\} \tag{23}\]

The main task of this section is to derive an adaptive law to adjust the regulating factor \( \alpha \) such that the estimated equivalent control \( u_f \) can be optimally approximated to the equivalent control of the SMC under the situations of unknown functions \( f_i \). For simplicity, a triangular type membership function is chosen for the aforementioned fuzzy variables. The on-line tuning algorithm of parameter is proposed to adjust the consequent parameters for monitoring the system control performance.

Suppose there exists an optimal regulating factor \( \alpha^* \) which is constant such that the \( u_f \) has minimum approximation error

\[e = u_f - u\] \tag{24}\]

Then from (19)-(21), we have

\[
\dot{u}_f - \dot{u} = \frac{1}{M} (\alpha - \alpha^*) \xi \tag{25}\]

where

\[\xi = w_i (i+j) + w_2 (i+j-1) + w_3 (i+j-1) + w_4 (i+j)\] \tag{26}\]

The control law for the FSMC with rule reduction system is assumed to make the following form:

\[
u = u_f + u_c \tag{27}\]

where \( u_f \) is the approximate equivalent control, and the correct control \( u_c \) is designed to stabilized the states of the control system around a pre-selected uncertainty bound. The following, substituting Eq. (27) into Eq. (1), we can obtain

\[
\dot{x} = f_i(x) + b_i(x) u = f_i(x) + b_i(x)(u_f + u_c - u_{eq}) \tag{28}\]

The above properties of the boundary layer concept are to be exploited, in the design of FSMC system, our goal being to case adaptation as soon as the boundary layer is reached. This approach aims to avoid the possibility of unbounded growth.

**Theorem 1:** Consider the dynamic system described by (1) and the decoupling sliding surface (2), for the bounded, continuous desired state trajectory with bounded velocity, if the FSMC law is designed as (27), in which the adaptation laws of the fuzzy controller are designed as (29) and the hitting controller is designed as (30) with estimation the unknown bound shown in (31), then can guarantee the asymptotic stability of the close-loop system. And the adaptive laws are given by

\[
\dot{\alpha} = -\dot{Y} = \frac{1}{M} \gamma s_i \xi \tag{29}\]

\[
u_c = -K \text{sat}(s_i / \Phi_i) \tag{30}\]
\[ \dot{K} = -\Xi = -\gamma_2 s_i \]  
where \( \eta_1 \) and \( \eta_2 \) are positive constants. Moreover, the system states converge to the sliding surface asymptotically.

**Proof:** Choose the Lyapunov function as
\[ V = \frac{1}{2h_1} s_i^2 + \frac{1}{2\gamma_1} \dot{Y}^2 + \frac{1}{2\gamma_2} \Xi^2 \]  
where \( Y = \alpha - \alpha^* \), \( \Xi = K - K^* \), and \( \gamma_1 \), \( \gamma_2 \) are positive constant.

The variation of this function (32) with respect to time is
\[
\dot{V} = \frac{s_i s_i}{h_1} + \frac{1}{2\gamma_1} \frac{\partial b_i^{-1}}{\partial x} x + \frac{1}{\gamma_1} \dot{Y} + \frac{1}{\gamma_2} \Xi
\]
\[
= -\frac{\lambda}{h_1} s_i^2 + s_i(u'_j - u_q) + \frac{1}{2} s_i \frac{\partial b_i^{-1}}{\partial x} x + s_i(u'_j - u_q') + s_i u_c
\]
\[
+ \frac{1}{\eta_1} (\alpha - \alpha^*) + \frac{1}{\eta_2} \Xi K
\]
\[
\leq -\frac{\lambda}{h_1} s_i^2 + s_i K^* + \frac{s_i}{\eta_1} (\alpha - \alpha^*) + \frac{s_i}{\eta_2} \Xi K
\]
\[
= -\frac{\lambda}{h_1} s_i^2 + \Xi (\frac{1}{\gamma_1} s_i K^* + \frac{1}{\gamma_2} (\frac{1}{M} + \frac{1}{\gamma_1} s_i) + \frac{1}{\gamma_2} (\frac{1}{M} + \frac{1}{\gamma_1} s_i))
\]  
(33)

By selecting appropriate values for \( \Phi_1 \), (33) implies \( \dot{V} \) is negative semidefinite
\[ \dot{V} \leq -\frac{\lambda}{h_1} s_i^2 \]  
(34)

In summary, the FSMC control law with the parameter vector \( \alpha \) adjusted by (29). The method can find appropriate fuzzy rules in fuzzy control implementation and on-line modification rule also has the effect of improving the stability property. An excellent style manual for science writers is [7].

**IV. COMPUTER SIMULATION RESULTS**

In this section, we shall demonstrate that the FSMC is applicable to a ball-beam system [12] to verify the theoretical development.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u + d \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= B(x_2^2 - G \sin x_1)
\end{align*}
\]  
(35)

where
\[
\begin{align*}
x_1 &= \theta \quad \text{is the angle of the pole with respect to the vertical axis}; \\
x_2 &= \dot{\theta} \quad \text{is the angle velocity of the pole with respect to the vertical axis}; \\
x_3 &= r \quad \text{is the position of the cart}; \\
x_4 &= r^* \quad \text{is the velocity of the cart}; \\
B &= \frac{MR^2}{J_b + MR}; \\
J_b &= \text{the moment of inertia of the ball}; \\
M &= \text{the mass of the ball}; \\
R &= \text{the radius of the ball}; \\
g &= \text{the acceleration of gravity}.
\end{align*}
\]

The center of rotation is assumed to be frictionless and ball is free to roll along the beam. It is required that the ball remains in contact with the beam and that rolling occurs without slipping. The objective is to keep the ball close to the center of the beam close to the horizontal position.

In the simulation, the following specifications are used: \( B = 0.7143 \), \( J_b = 2 \times 10^{-4} \), \( M = 0.05 \text{kg} \), \( R = 0.01 \text{m} \), \( g = 9.8 \text{m/s}^2 \), \(|d| \leq 0.08 \), \( c_1 = 5 \), \( c_2 = 0.5 \), \( \Phi_1 = 5 \), \( \Phi_2 = 5 \), \( Z_u = 0.9425 \), \( \gamma_1 = 2 \), \( \gamma_2 = 2 \).

Initial values are
\[
\begin{align*}
x_1 &= \theta = 60^o, \\
x_2 &= \dot{\theta} = 0^o, \\
x_3 &= 10, \\
x_4 &= r = 0.4.
\end{align*}
\]

Fig. 2 through Fig. 6 shows the simulation results. It is found that the ball-beam system can be stabilized to the equilibrium point, and shown that \( q \) and \( r \) converge to zero, respectively. Further, the proposed control performance and robustness better [12].

**Figure 2. Position evolution of the ball.**
The fuzzy rules of FSMC should be preconstructed by trial-and-error tuning. By the FSMC, the fuzzy rules can be learned on-line by rule reduction, and the stability of the proposed FSMC system can be guaranteed. Its learning scheme can be started with zero initial fuzzy rules and converged quickly without the requirement of mathematical model. This control strategy establishes the appropriate fuzzy rules by continuous on-line learning instead of trial-and-error process. The simulated results of system have been provided to demonstrate the effective of the proposed control systems.

REFERENCES