Hybrid Grey Fuzzy Sliding Mode Control for Seesaw Systems

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Abstract

In this paper, we will propose a signed distance grey fuzzy sliding mode controller (SDGFSMC) is implemented on the Texas Instruments TMS320LF2407A digital signal processor (DSP). The feature of the SDGFSMC is that it only needs a single grey fuzzy input. The advantage of the approach is that the total number of rules ha been greatly reduced comparing with the conventional fuzzy control. Due to the limitation of computation ability, the low-cost general-purpose micro-controller or microprocessors is difficult to implement advanced real-time algorithms in for industrial applications. The simulation and experiment Seesaw system result show which the proposed controller shown that the proposed method is feasible and effective.

Keywords: Fuzzy, Sliding mode control, Grey, DSP

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1. Introduction

The grey theory applicable to the prediction problem of a time-varying nonlinear system, was first proposed by Deng in 1982 [25]. Currently, it has been widely and successfully used in many fields, such as economics, history, geography, medicine, traffic, weather, and automatic control [21-23]. In the grey system, the information, such as operation mechanism, structure and behaviour, is neither deterministic (white) nor totally unknown (black), but is partially known (grey). Instead of forming a knowledge base, the grey model constructs some differential equations to characterize the controlled system behaviour. By using a few past output data and solving the differential equations, the grey model usually predicts the next output of the system accurately.

It is known that the design of a controller usually requires an explicit mathematical model of the controlled plant, if a conventional design method is used [2]. However, a complete mathematical model of a plant such as, nonlinear dynamic systems cannot be easily obtained. Therefore, more and more advanced control methods have been developed for these systems with time-varying parameters, such as fuzzy control, adaptive control, robust control, neural network, etc [1-6].

In recent years, fuzzy logic with grey predictor has been a popular method for designing a controller. The grey fuzzy controller has the robustness and requires minimal modeling of the plant and is simpler in design [15-19]. These advantage and other properties make fuzzy logic become a promising method for designing controllers. Unfortunately, fuzzy controllers have several problems. Two of the main problems with such controllers lie in assuring the stability of the closed-loop system and in designing fuzzy controllers for high-order systems. To overcome these problems, several authors have proposed design methods for fuzzy controllers. This
has resulted in the area termed fuzzy sliding mode control (FSMC). With FSMC, a Variable Structure System controller is designed with guaranteed stability properties [6-8]. On the other hand, the advantage of such controllers is that the number of rules required is reduced from $m^4$ to $nm^2$ in [6] or $m^2$ in [1]. In this paper, we proposed the SDFSMC control with grey predictor. This controller has guaranteed stability properties and it needs only one-dimensional space as its input space. In this paper, we adopt a digital signal processor to implement the SDGFSMC. There are many advantages in using DSP to implement control algorithm. In addition, the SDGFSMC is an uncomplicated control problem and suitable for DSP implementation particularly [8-11].

This controller guarantees some properties, such as the robust performance and stability properties. We show that the SDGFSMC has the following advantages: (1) It can well control most of complex systems without knowing their exact mathematical models. (2) The dynamic behavior of the controlled system can be approximately dominated by a sliding surface. (3) SDGFSMC can not only increase the robustness to system uncertainties but also decrease the chattering phenomenon in the conventional sliding mode controller.

The organization of this paper is as follows. In Section 2, we introduce the signed distance fuzzy sliding mode controller. In Section 3, the grey theory are described. In Section 4, we discuss the implementation of SDGFSMC via DSP method. In Section 5, we show the simulation and the experiment result. Finally, we conclude with Section 6.
2. System description and fuzzy sliding mode control

2.1. System description

Consider an $n^{th}$-order nonlinear system described by the following state-space model in a canonical form:

\[
x_1(t) = x_2(t), \\
x_2(t) = x_3(t), \\
\vdots \\
x_n(t) = f(x) + b(x)u + d(t), \\
y(t) = x_1(t), \text{ for } t \geq 0,
\]

where $x = [x_1, x_2, \cdots, x_n]^T$ is the state vector, $u$ is the control input, $f(x)$ and $b(x)$ are nonlinear functions, $y(t)$ is the system output, and $d(t)$ represents the external disturbance. The superscript $T$ stands for the transpose of matrix. If the reference input $y_r(t)$ is a step function, then the above dynamic equations can be transformed into the following state equations with error signal $e = y_r - y$ and its derivatives as state variables:

\[
\dot{e}_1(t) = e_2(t), \\
\dot{e}_2(t) = e_3(t), \\
\vdots \\
\dot{e}_n(t) = -f(e) - b(e)u - d(t), \text{ for } t \geq 0.
\]

Let $e = [e_1, e_2, \cdots, e_n]^T \in \mathbb{R}^n$ and $\delta \in \mathbb{R}$. A linear function $s : e \rightarrow \delta$ is defined by

\[
S(e) = ce,
\]

where $c = [c_1, c_2, \cdots, c_n, 1]$, $c \in \mathbb{R}^n$. Then, a sliding hyperplane can be represented as

\[
S = 0 \quad \text{or} \quad S = c_1e_1 + c_2e_2 + \cdots + c_ne_n = 0
\]

The control purpose is to force the trajectory $e$ to reach the sliding surface $S$ in finite time and eventually sliding toward the origin $e = 0$.

Based on the sliding mode control principle, the objective is to design a control law $u$ such that the reaching condition:

\[
\dot{S}S < 0,
\]
is satisfied, where \( \dot{S} \) represents the time derivative of \( S \).

### 2.2. Signed distance fuzzy sliding-mode control

In this section, the idea of named the signed distance is used, and the feasibility of the present approach will be demonstrated. The switching line is defined by:

\[
S: \dot{e} + c_i e = 0
\]  

(6)

First, we introduce a new variable called the signed distance. Let \( A(e, \dot{e}) \) be the intersection point of the switching line and the line perpendicular to the switching line from an operating point \( B(e, \dot{e}) \), as illustrated in Fig. 1. Next, \( d \) is evaluated. The distance between \( A(e, \dot{e}) \) and \( B(e, \dot{e}) \) can be given by the following expression [7]:

\[
d = \sqrt{[(e - e_c)^2 + (\dot{e} - \dot{e}_c)^2]} = \frac{\|e + c_i e\|}{\sqrt{1 + c_i^2}}
\]

(7)

Without loss of generality, Eq. (7) can be rewritten as follows:

\[
d = \frac{|e + c_i e|}{\sqrt{1 + c_i^2}}
\]

(8)

The signed distance \( d_s \) is defined for an arbitrary point \( B(e, \dot{e}) \) as follows:

\[
d_s = \text{sgn}(S) \frac{\|e + c_i e\|}{\sqrt{1 + c_i^2}} = \frac{e + c_i e}{\sqrt{1 + c_i^2}} = \frac{S}{\sqrt{1 + c_i^2}}
\]

(9)

where

\[
\text{sgn}(S) = \begin{cases} 
1 & \text{for } S > 0 \\
-1 & \text{for } S < 0 
\end{cases}
\]

(10)

Now, we choose a Lyapunov function:

\[
V = \frac{1}{2} d_s^2
\]

(11)

Then

\[
\dot{V} = d_s \dot{d}_s = \frac{S \dot{S}}{1 + c_i^2}
\]

(12)

Hence, it is seen that if \( S > 0 \), then \( d_s > 0 \), decreasing \( u \) will make \( S \dot{S} \) decrease.
so that \( \dot{V} < 0 \) and that if \( S < 0 \), then \( d_s < 0 \), increasing \( u \) will make \( S \) decrease so that \( \dot{V} < 0 \). So we can ensure that the system is asymptotically stable. From the above relation, we can conclude that:

\[
u \propto - d_s
\]

(13)

Hence, the fuzzy rule table can be established on a one-dimensional space of \( d_s \) as shown in Table I instead of a two-dimensional space of \( x \) and \( \dot{x} \). The control action can be determined by \( d_s \) only. Hence, we can easily add or modify rules for fine control. For implementation, a triangular type membership function is chosen for the aforementioned fuzzy variables, as shown in Fig. 2. However, \( d_s \) and \( u \) are the input and output of the signed distance fuzzy logic control, respectively. The input of the proposed fuzzy controller is a fuzzified variable of \( d_s \). The output of the fuzzy controller which is the fuzzified variable of \( u \) and all the universes of discourse of \( d_s \) and \( u \) are arranged from –1 to 1. The SDFSMC is shown in Fig.3, where \( S \) is the input of the SDFSMC and \( u \) denotes the output of the SDFSMC.

3. Grey Theory

Grey system is a novel scientific theory and is first proposed by Professor Deng Julong in the 1982. It, mainly, works on a system analysis with poor, incomplete, or uncertain messages [12-14]. Particularly the single-variable first-order differential equation is used to model the GM(1,1) that only uses a few data for modeling process. The GM(1,1) model is defined as

\[
\frac{dx^{(1)}(k)}{dk} + ax^{(1)}(k) = b
\]

(14)

In the modeling procedure of GM, the original data are preprocessed by using the Accumulated Generating Operation (AGO) for getting the information of the
modeling and decreasing the random behavior of system, and then take the generated
data to construct the grey model. The modeling procedure is given below.

Step 1. Let the original data is \( x^{(0)} \)
\[
x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))
\]

(15)

Step 2. Let \( x^{(1)} \) be the one time AGO (1-AGO) of the \( x^{(0)} \)
\[
x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))
\]

where \( x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m) \quad k = 1, 2, \ldots, n \)

(17)

Step 3. Using the least square method to calculate the model parameters \( \hat{a}, \)
\[
\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} y_n
\]

which can be expressed with in the following matrix form:
\[
y_n = B \hat{\theta}
\]

(19)

where
\[
y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad \hat{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}
\]
\[
z^{(1)}(k) = \frac{1}{2} x^{(0)}(k) + \frac{1}{2} x^{(0)}(k-1), \quad \forall k = 2, 3, \ldots, n
\]

(20)

Step 4. The predictive function is obtained
\[
x^{(0)}(k) + az^{(1)}(k) = b, \quad \forall k = 2, 3, \ldots, n
\]

(21)

where \( a \) is the development coefficient and \( b \) is the grey control quantity.

Therefore, the solution can be written as
\[
x^{(1)}(n + p) = \frac{b}{a} + \frac{b}{a}
\]

(22)

where
\[ c = (y^{(0)}(1) - \frac{b}{d}) \cdot e^a \]  

(23)

Taking the inverse the inverse accumulated generating operation (IAGO) to 
\( x^{(1)}(n + p) \), then the predictive value of the original sequence will be obtained by

\[ x^{(0)}(n + p) = c \cdot e^{-a(n + p)} - c \cdot e^{-a(n + p - 1)} \]  

(24)

Finally, the parameters of (20) are obtained by the least square approximation method as

\[
\begin{bmatrix}
  a \\
  b
\end{bmatrix} = (B^T B)^{-1} B^T y_n
\]

Based on the above description, the grey predictor composed of AGO, IAGO and 
GM(1,1) can be constructed by

\[ x^{(0)} = IAGO \circ GM(1,1) \circ AGO \circ x^{(0)} \]  

(25)

According to the grey prediction, all the elements in the original sequence \( x^{(0)} \) should be non-negative values, so we have to transform \( y^{(0)} \) to the relative positive-sequence by the mapped generating operation (MGO). Oppositely, we can also use the inverse operation (IMGO) to get the original sequence. The MGO operator is expressed as

\[ x^{(0)}_m = MGO \circ x^{(0)} \circ x^{(0)} + \Delta \]  

(26)

where

\[ \Delta = \min(y^{(0)}) + \varepsilon \]  

(27)

In addition, its inverse operator (IMGO) is

\[ x^{(0)} = IMGO \circ x^{(0)} \circ x^{(0)} - \Delta \]  

(28)

Hence the modified grey predictor can be constructed by

\[ x^{(0)} = IMGO \circ IAGO \circ GM(1,1) \circ MEAN \circ AGO \circ MGO \circ x^{(0)} \]  

(29)

Define a performance index as
\[ J = \sum_{k=1}^{n} (\hat{y}^{(0)} (k) - y^{(0)}(k))^2 \]

(30)

\[ = (y^{(0)} (1) - y^{(0)}(1))^2 + \sum_{k=2}^{n} (\hat{y}^{(0)} (k) - y^{(0)}(k))^2 \]

where \( \hat{y}^{(0)}(k) \) is prediction result.

The new predicted values [23] are given as

\[ y^{(0)} (1) = Ce^{-a} + \frac{u}{a} \]

and

\[ y^{(0)}(k) = C(e^{-ka} - e^{-(k-1)a}), \forall k = 2,3,...,n. \]

\[ C = \frac{(y^{(0)} (1) - \frac{u}{a} e^{-a} + \sum_{k=2}^{n} y^{(0)}(k) e^{-(k-1)a} (e^{-a} - 1)}{e^{-2na} + \frac{2e^{-2a}}{e^{-2a} + (e^{-a} + 1)}}(1 - e^{-2(n-1)a}) \]

(31)

where \( C \) minimizes the performance index in (30)

4. The Grey Signed Distance Fuzzy Sliding Mode Controller

In this method, the grey predictor yields two forecasting values at one time. They are different depending on their prediction mode. A fuzzy inference scheme is used to alter the mode. A fuzzy inference scheme is used to alter the modes in accordance with the errors in the system. The fuzzy scheme is performed like a switch device. So we call it as a Fuzzy-Switching device. In addition, we add a fuzzy controller to product an offset value.

The \( \hat{e}_+ \) and \( \hat{e}_- \) are resulted from the grey predictor according to the different steps \( G_+ \) and \( G_- \), i.e. the positive and negative step. The prediction results (\( \hat{e}_+ \) and \( \hat{e}_- \)) are fed into the consequence part in the fuzzy scheme. The premise part depends
on the system errors to switch the forecasted modes. The practical value $\hat{e}$ is decided and transmitted into a signed distance fuzzy sliding-mode controller to yield the control signal $u$.

\[
\begin{align*}
\hat{e}^{(0)}_+ (n + p) &= C \cdot e^{-a(n+p)} - C \cdot e^{-a(n+p,-1)} - \Delta, \quad \text{Positive Step} \ (G_+) \quad (32)
\hat{e}^{(0)}_- (n + p) &= C \cdot e^{a(n+p)} - C \cdot e^{a(n+p,-1)} - \Delta, \quad \text{Negative Step} \ (G_-) \quad (33)
\end{align*}
\]

where

\[
C = \frac{(e_m^{(0)}(1) - \frac{\mu}{a})e^{-a} + \sum_{k=2}^{n} y^{(0)}(k)e^{-(k-1)a}(e^{-a} - 1)}{e^{-2na} + \frac{2e^{-2a}}{(e^{-a} + 1)}(1 - e^{-2(n-1)a})} \quad (34)
\]

In this section, $e_m^{(0)}(1)$ is the first element in the MGO sequence of the input sequence $e(k)$. By means of fuzzy theory, we define the membership functions of the premise and the consequence part of fuzzy switching as follows:

There are only two fuzzy sets in each linguistic variable in Fig. 5. The fuzzy If-Then rule statements are defined by

- **Rule 1**: If error is large, then prediction mode is negative-step prediction
- **Rule 2**: If error is small, then prediction mode is negative-step prediction

The **Rule 1** performs the negative-step forecasted mode to reduce the rise time, and **Rule 2** performs the positive-step mode to prevent the system overshoot. The defuzzification is determined as

\[
e = \frac{\mu_{\text{small}}(e)e_+ + \mu_{\text{large}}(e)e_-}{\mu_{\text{small}}(e) + \mu_{\text{large}}(e)} \quad (35)
\]

If error is located in the range of $[-1, E_s]$ and $[E_t, 1]$, then one rule is fired. There is just one mode used to forecast the system error. But if the error is in $(E_s, E_t)$ then both the rules are turned on; the predicted result is a compensation of the distinct prediction modes. The defuzzification in Fig. 6 is accomplished by the
weight-average method presented in Eq. (35). The $\mu_{\text{small}}(e)$ and $\mu_{\text{large}}(e)$ are denoted as the fired weight in the fuzzy rules. Furthermore, the THEN-part memberships are altered following the prediction values every sampling time.

5. Simulation and Experimental Result

In this section, we shall demonstrate that the SDGFSMC design is applicable to the seesaw system to verify the theoretical development. A DSP-based real-time control EVM module was used to implement the proposed SDGFSMC system. The EVM module has a Texas Instruments TMS320 LF2407A central processing unit, analog-to-digital and digital-to-analog converters, PWM generators and I/O etc. The TMS320LF2407A is a 16 bit fixed-point processor with 25-ns instruction execution time. The sampling frequency for experimental implementation of the proposed SDGFSMC drive system is 8 kHz.

The previous study gives the system model by using Lagrange’s formulations based on principle of balance of force and torque below.

\[
\begin{align*}
    m(r_1 \ddot{\theta} + x) - mx \dot{\theta}^2 - mg \sin \theta &= u, \\
    I \ddot{\theta} + m[r_1 (r_1 \dot{\theta} + x) + x^2 \ddot{\theta} + 2xx \dot{\theta}] - Mr_1 \sin \theta - mg(r_1 \sin \theta + x \cos \theta) &= 0
\end{align*}
\]

The dynamical equation of the seesaw mechanism is given as follows:

\[
\begin{align*}
    u + mg \sin \theta - B \dot{x} &= mx, \\
    (Mg \sin \theta) r_1 + mg \sin (\theta + \phi) \cdot \sqrt{(x^2 + r_1^2)} + ur_1 - \mu \dot{\theta} &= I \ddot{\theta}
\end{align*}
\]

where $I$ is the wedge inertia given by (32)

\[
I = M \left( \frac{a^2 + \frac{r_1^2}{2}}{24} \right)
\]

From Fig. 9, it can be derived below.
\[ I = \rho c \int \int (x^2 + y^2) \, dx \, dy \]
\[ = \rho c \int_0^b \int_{\frac{a}{2}}^{\frac{b}{2}} (x^2 + y^2) \, dx \, dy \]
\[ = \frac{1}{2} \rho abc \left( \frac{a}{24} + \frac{b^2}{2} \right) \quad (39) \]

5.1. The Result of Computer Simulation

In order to prove the practicability of SDGFSMC, simulation is done before it is applied to the practical system. The initial states of these two simulations are different. The constants \([c_1, c_2, c_3]\) selected are \([1.5, 2.3, 6.2]\). For the choice of coefficients, they are selected by trial-and-error and are determined under the condition of asymptotic stability.

5.2 The Application Algorithm

For the seesaw system given in (36), the seesaw system has three variables of states. The variables are the angle that the wedge makes with the vertical line, change of the angle that the wedge makes with vertical line, and the position of the cart from the origin.

Define the sliding surface
\[ S = c_1 e_1 + c_2 e_2 + c_3 e_3 \quad (40) \]
where \([e_1, e_2, e_3] = \begin{bmatrix} 0 - \theta & 0 - \dot{\theta} & 0 - x \end{bmatrix} \]

The Fig. 14 is the practical hardware structure of the seesaw system. In Experiment 1 and Experiment 2 have dissimilar initial states. Disturbance is added in Experiment 3, when the seesaw system reaches balance. The constants \([c_1, c_2, c_3]\) selected are \([1.8, 2.7, 7.5]\), the choice of coefficients are selected by trial-and-error and are determined under the condition of asymptotic stability.

**Experiment 1:**
The initial states of the seesaw:

The position of the cart \( x \) is 37 centimeters.

The angle of inverted wedge \( \theta \) is 14.0 degrees.

**Experiment 2:**

The initial states of the seesaw:

The position of the cart \( x \) is -34 centimeters.

The angle of inverted wedge \( \theta \) is -14.0 degrees.

Based on what we want to balance the angle of inverted wedge and the position of cart, we can give the proper commands for balancing control in different situations, and thus construct the controller of the system.

In the results of experiment 1 and experiment 2, once the states reach the sliding surface of the state space, they stay on the sliding surface. Similarly, it is found that the responses of \( S \), position and angle own the same feature under different initial states. The major advantages of the SDGFSMC are the rule reduction and robustness, as well as without the exact mathematical model. Fig. 8 and Fig. 9 of experiment 1 show the response performance of SDGFSMC where the initial value of is 37cm, is 14° and the settling time is 5.3s. Figs. 12 and Fig. 13 of experiment 2 show the response performance of SDGFSMC where the initial value of is 34cm, is -14° and the settling time is 5.1s. Figs. 24 and Fig. 26 of experiment 3 show the response performance of SDGFSMC where the initial value of is 37cm, is 14°, the disturbance is added at \( t = 8s \) and the settling time is 10.3s.

6. Conclusions
A DSP real-time implementation method in SDGFSMC is proposed in this paper. From the above description, we have noticed that the SDGFSMC method is an uncomplicated control algorithm and suitable for DSP implementation particularly. The proposed method is applied in controlling a seesaw system which has been proposed to solve the position balance control problem for highly nonlinear and coupling, and the complete dynamic model is difficult to obtain precisely. Chattering and the excessive activity of the control signal are eliminated without a degradation of the trajectory following performance. To verify the effectiveness of the proposed control scheme, the SDGFSMC control system was implemented to control a seesaw system. From the simulation results show the seesaw system will be arrived at purpose, and the seesaw system can be stabilized to the equilibrium.
References


Manufacture, 42, 343-355.


Table I. Rule table for SDGFSMC

<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZE</th>
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<tr>
<td>$u$</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>ZE</td>
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![Diagram](image)

Fig. 1. Derivation of a signed distance.
Fig. 2. Fuzzy variable of triangular type.

Fig. 3. The block diagram of the SDFSMC.
Fig. 4. The block diagram of the SDGFSMC controller.

Fig. 5. The membership function in the premise part of the fuzzy-switching scheme.
Fig. 6. The membership function in the consequence part of the fuzzy-switching scheme.

Fig. 7. The practical hardware structure of the seesaw system.
Fig. 8. The initial states of the seesaw of experiment 1.

Fig. 9. The position response of the practical seesaw system of experiment 1.
Fig. 10. The angle response of the practical seesaw system of experiment 1.

Fig. 11. The initial states of the seesaw of experiment 2.
Fig. 12. The position response of the practical seesaw system of experiment 2.

Fig. 13. The angle response of the practical seesaw system of experiment 2.