



Chapter 2

Boolean Algebra and Logic Gates

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Outline

- Basic Definitions
- Basic Theorems and Properties
- Boolean Functions
- Canonical and Standard Forms
- Digital Logic Gates
- Integrated Circuits

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Basic Definition

Boolean Algebra:

- A deductive mathematical system
- Defined with
 - A set of elements
 - ex: $B = \{0, 1\}$
 - A set of operators
 - ex: $+$, $*$, ...
 - A number of unproved axioms or postulates

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Most Common Postulates (1/2)

- Closure: (封閉空間)
 - The result of any operators is still in a set S if all operands are also in set S
 - ex: natural number $N = \{1, 2, 3, \dots\}$
natural $+$ natural = natural
- Associate law: (結合律)
 - ex: $(X * Y) * Z = X * (Y * Z)$ for all X, Y, Z in set S
- Commutative law: (交換律)
 - ex: $X * Y = Y * X$ for all X, Y in set S

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Most Common Postulates (2/2)

- Identity element: (單位元素)
 - If e is an identity element w.r.t an operator $*$, then
$$e * X = X * e = X$$
 for every X in set S
 - ex: $x + 0 = 0 + x = x$ (x is an integer)
- Inverse: (反元素)
 - If I is an inverse of x w.r.t an operator $*$, then
$$x * I = e$$
 - ex: $a + (-a) = 0$ (0 is the identity element of $+$)
- Distributive law: (分配律)
 - ex: $X * (Y + Z) = X * Y + X * Z$ ($*$ over $+$)

P.S: w.r.t = with respect to

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History of Boolean Algebra

- In 1854, George Boole
 - Introduced a systematic treatment of logic
 - Developed an algebraic system called ***Boolean algebra***
- In 1938, C. E. Shannon
 - Introduced a two-valued Boolean algebra called ***switching algebra***
 - The properties of bistable electrical switching circuits (digital circuits) can be represented by it
- In 1904, E. V. Huntington
 - Formulate the postulates as the formal definitions

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Postulates of Boolean Algebra

Boolean algebra is

- Defined by a set of elements ***B***
- Defined with two binary operators
 - ***+*** (***OR***), ******* (***AND***)
- Satisfies the following postulates:
 - (1) Closure w.r.t the operators ***+*** and *******
 - (2) Identity w.r.t ***+*** is 0; identity w.r.t ******* is 1
 - (3) Commutative w.r.t ***+*** and *******
 - (4) Distributive w.r.t ***+*** and *******
 - (5) $x + x'$ (complement) = 1; $x * x' = 0$
 - (6) There exists at least two different elements in ***B***

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Boolean v.s. Ordinary

- | | |
|--|---|
| <ul style="list-style-type: none">■ Boolean Algebra<ul style="list-style-type: none">▪ Associate law not included (but still valid)▪ Distributive law is valid▪ No additive or multiplicative inverses▪ Define <i>complement</i> in postulate 5▪ No. of elements is not clearly defined<ul style="list-style-type: none">▪ 2 for two-valued Boolean algebra | <ul style="list-style-type: none">■ Ordinary Algebra<ul style="list-style-type: none">▪ Associate law included▪ Distributive law may not valid▪ Have additive and multiplicative inverses▪ No complement operator▪ Deal with real numbers<ul style="list-style-type: none">▪ Infinite set of elements |
|--|---|

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Two-Valued Boolean Algebra

A two-valued Boolean algebra is

- Defined on a set of two elements $B = \{0, 1\}$
- With rules for the binary operators $+$ and $*$

x	y	$x*y$
0	0	0
0	1	0
1	0	0
1	1	1

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

x	x'
0	1
1	0

- Satisfying the six Huntington postulates
 - Proofs are shown in the text book
- Called “switching algebra”, “**binary logic**”

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Duality Principle

- Every Boolean algebraic expression remains valid if the **operators** and **identity elements** are interchanged
 - Part (a) and part (b) are dual in the Huntington postulates
- For two-valued Boolean algebra:
 - Interchange OR and AND operators and replace 1's by 0's and 0's by 1's
 - Ex: $X + 1 = 1 \rightarrow X * 0 = 0$

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Postulates of Binary Logic

- Postulate 2:
 - (a) $X + 0 = X$ (b) $X * 1 = X$
- Postulate 3: (commutative)
 - (a) $X + Y = Y + X$ (b) $X * Y = Y * X$
- Postulate 4: (distributive)
 - (a) $X(Y + Z) = XY + XZ$
 - (b) $X + YZ = (X + Y)(X + Z)$
- Postulate 5:
 - (a) $X + X' = 1$ (b) $X * X' = 0$

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Theorems of Binary Logic (1/6)

- Theorem 1(a)

$$X + X = X$$

<proof>

$$\begin{aligned} X + X &= (X + X) * 1 \dots\dots\dots p2(b) \\ &= (X + X) (X + X') \dots p5(a) \\ &= X + XX' \dots\dots\dots p4(b) \\ &= X + 0 \dots\dots\dots P5(b) \\ &= X \dots\dots\dots P2(a) \end{aligned}$$

- Theorem 1(b)

$$X * X = X$$

<proof>

$$\begin{aligned} X * X &= XX + 0 \dots\dots\dots p2(a) \\ &= XX + XX' \dots\dots\dots p5(b) \\ &= X (X + X') \dots\dots\dots p4(a) \\ &= X * 1 \dots\dots\dots p5(a) \\ &= X \dots\dots\dots p2(a) \end{aligned}$$

Theorems of Binary Logic (2/6)

- Theorem 2(a): $X + 1 = 1$

<proof>

$$\begin{aligned} X + 1 &= 1 * (X + 1) \dots\dots\dots p2(b) \\ &= (X + X') (X + 1) \dots\dots\dots p5(a) \\ &= X + X' * 1 \dots\dots\dots p4(b) \\ &= X + X' \dots\dots\dots p2(b) \\ &= 1 \dots\dots\dots p5(a) \end{aligned}$$

- Theorem 2(b): $X * 0 = 0$

<proof>

by duality of theorem 2(a)

Theorems of Binary Logic (3/6)

- Theorem 3: $(X')' = X$ (involution, 乘方)

<proof>

$$X + X' = 1 \text{ and } X * X' = 0 \text{ (from p5)}$$

\Rightarrow X and X' are complement to each other

\therefore the complement of $X' = (X')' = X$

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Theorems of Binary Logic (4/6)

- Theorem 4: (associative)

(a) $X + (Y + Z) = (X + Y) + Z$

(b) $X (YZ) = (XY) Z$

<proof for (a)>

$$\begin{aligned} X + (Y + Z) &= X*1 + (Y + Z)*1 \dots\dots\dots \text{p2(b)} \\ &= X*1 + Y*1 + Z*1 \dots\dots\dots \text{p4(a)} \\ &= (X + Y)*1 + Z*1 \dots\dots\dots \text{p4(a)} \\ &= (X + Y) + Z \dots\dots\dots \text{p2(b)} \end{aligned}$$

<proof for (b)>

can be obtained by the duality of theorem 4(a)

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Theorems of Binary Logic (5/6)

- Theorem 5: (DeMorgan)

$$(a) (X + Y)' = X'Y' \quad (b) (XY)' = X' + Y'$$

<proof>

(a) by truth table

x	y	x+y	(x+y)'	x'	y'	x'y'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

(b) can be proofed by similar way

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Theorems of Binary Logic (6/6)

- Theorem 6: (absorption, 合併)

$$(a) X + XY = X \quad (b) X(X + Y) = X$$

<proof for (a)>

$$\begin{aligned} X + XY &= X * 1 + XY \quad \dots\dots\dots p2(b) \\ &= X(1 + Y) \quad \dots\dots\dots p4(a) \\ &= X(Y + 1) \quad \dots\dots\dots p3(a) \\ &= X * 1 \quad \dots\dots\dots p2(a) \\ &= X \quad \dots\dots\dots p2(b) \end{aligned}$$

<proof for (b)>

can be obtained by the duality of theorem 6(a)

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Operator Precedence

- (1) Parentheses (括號)
- (2) NOT similar to the sign of numbers
- (3) AND similar to multiplication
- (4) OR similar to addition

Ex: $(X + Y)' + Z$

step 1: $X + Y$ inside the parentheses

step 2: $(X + Y)'$

step 3: $(X + Y)' + Z$

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Boolean Functions

- A Boolean function is described by an algebraic expression that consists of
 - Binary variables
 - The constant 0 and 1
 - The logic operation symbols
- For a given value of the binary variables, the function can be equal to either 1 or 0
- Ex: $F1 = X + Y' Z$
 - $X = 1 \rightarrow F1 = 1$
 - $Y = 0$ and $Z = 1 \rightarrow F1 = 1$
 - otherwise $\rightarrow F1 = 0$

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Truth Tables

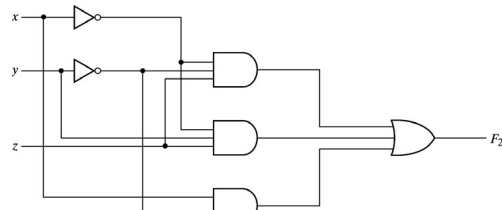
- A Boolean function can be represented in a ***truth table***
 - An unique representation
- A truth table includes
 - A list of combinations of 1's and 0's assigned to the binary variables
 - A column that shows the value of the function for each binary combination
- Ex: $F1 = X + Y' Z$
 - No. of binary variables = 3
 - No. of rows = $2^3 = 8$

x	y	z	F1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

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Gate Implementation

- A Boolean function can be transformed from an algebraic expression into a circuit diagram composed of logic gates
- The forms of the algebraic expressions have large impact on the logic circuit diagram
 - Not an unique representation



(a) $F_2 = x'y'z + x'yz + xy'$



(b) $F_2 = xy' + x'z$

Fig. 2-2 Implementation of Boolean function F_2 with gates

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Algebraic Manipulation

- Literal: a single variable within a term that may be complement or not
 - $F_2 = x'y'z + x'yz + xy'$ (3 terms, 8 literals)
 - $F_2 = xy' + x'z$ (2 terms, 4 literals)
- Reducing the **number of terms** and the **number of literals** can often lead to a simpler circuit
- Simplify methods:
 - By map methods described in Chapter 3 (up to 5 variables)
 - By computer minimization programs
 - By a cut-and-try procedure employing the algebraic manipulation techniques (the only manual method)

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Example 2-1

- Simplify the following Boolean functions to a minimum number of literals
 1. $x(x' + y) = xx' + xy = 0 + xy = xy$
 2. $x + x'y = (x + x')(x + y) = 1(x + y) = x + y$
 3. $(x + y)(x + y') = x + xy + xy' + yy'$
 $= x(1 + y + y') + 0 = x$
 4. $xy + x'z + yz = xy + x'z + yz(x + x')$
 $= xy + x'z + xyz + x'yz$
 $= xy(1 + z) + x'z(1 + y) = xy + x'z$
 5. $(x + y)(x' + z)(y + z) = (x + y)(x' + z)$
by duality from function 4

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Complement of a Function

- Can be derived algebraically through DeMorgan's theorem
 - The basic theorem 5 for two variables
- DeMorgan's theorem can be extended to three or more variables
 - $(A + B + C + \dots + F)' = A'B'C' \dots F'$
 - $(ABC \dots F)' = A' + B' + C' + \dots + F'$
 - Proof for three variables is shown in the text book
- The complement of a function is

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Example 2-2 & 2-3

- 2-2: complement by DeMorgan's theorem

$$\begin{aligned}F1' &= (x'yz' + x'y'z)' = (x'yz')'(x'y'z)' \\ &= (x + y' + z)(x + y + z')\end{aligned}$$

$$\begin{aligned}F2' &= [x(y'z' + yz)]' = x' + (y'z' + yz)' \\ &= x' + (y'z')'(yz)' = x' + (y + z)(y' + z')\end{aligned}$$

- 2-3: complement by taking duals and complementing each literal

$$\begin{aligned}F1 &= x'yz' + x'y'z \quad (\text{dual}) \rightarrow (x' + y + z')(x' + y' + z) \\ (\text{complement}) &\rightarrow (x + y' + z)(x + y + z') = F1'\end{aligned}$$

$$\begin{aligned}F2 &= x(y'z' + yz) \quad (\text{dual}) \rightarrow x + (y' + z')(y + z) \\ (\text{complement}) &\rightarrow x' + (y + z)(y' + z') = F2'\end{aligned}$$

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Minterms and Maxterms

x	y	z	Minterms		Maxterms	
			Term	Name	Term	Name
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterm = standard product

Maxterm = standard sum

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Canonical Form

- $$F1 = x'y'z + xy'z' + xyz$$

$$= m_1 + m_4 + m_7$$

- $$F1' = x'y'z' + x'yz' + x'yz + xy'z' + xyz'$$

$$\rightarrow F1 = (x+y+z)(x+y'+z)(x+y'+z')$$

$$(x'+y+z')(x'+y'+z)$$

$$= M_0 M_2 M_3 M_5 M_6$$

x	y	z	F1	F2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

- Similarly:

$$F2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

$$= (x+y+z)(x+y+z')(x+y'+z)(x'+y+z) = M_0 M_1 M_2 M_4$$

- Boolean functions expressed as a sum of minterms or product of maxterms are said to be in **canonical form**

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Sum of Minterms

- Each term must contain all the variables
 - If x is missing, ANDed with $(x + x')$
 - Example 2-4:
 - Express $F = A + B'C$ in a sum of minterms
 - First term is missing two variables (B, C):

$$A = A(B+B') = AB+AB' = AB(C+C') + AB'(C+C')$$

$$= ABC+ABC'+AB'C+AB'C'$$
 - Second term is missing one variable (A):

$$B'C = B'C(A+A') = AB'C + A'B'C$$
- $\Rightarrow F = A+B'C = ABC+ABC'+AB'C(\text{twice})+AB'C'+A'B'C$
- $= m_1+m_4+m_5+m_6+m_7$

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Notation for Sum of Minterms

- $F = A+B'C = ABC+ABC'+AB'C+AB'C'+A'B'C$
- $= m_1+m_4+m_5+m_6+m_7$
- $\Rightarrow F(A, B, C) = \Sigma(1,4,5,6,7)$
 - Σ : ORing of terms
- Can be derived directly from the truth table

A	B	C	F1	
0	0	0	0	
0	0	1	1	$\rightarrow m_1$
0	1	0	0	
0	1	1	0	
1	0	0	1	$\rightarrow m_4$
1	0	1	1	$\rightarrow m_5$
1	1	0	1	$\rightarrow m_6$
1	1	1	1	$\rightarrow m_7$

$\rightarrow \Sigma(1,4,5,6,7)$

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Product of Maxterms

- Each term must contain all the variables
 - If x is missing, ORed with xx' and apply distributive law
- Example 2-5:
 - Express $F = xy + x'z$ in a product of maxterms

$$F = xy + x'z = (xy + x')(xy + z) = (x + x')(y + x')(x + z)(y + z)$$

$$= (x' + y)(x + z)(y + z)$$

$$x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$$

$$x + z = x + z + yy' = (x + y + z)(x + y' + z)$$

$$y + z = y + z + xx' = (x + y + z)(x' + y + z)$$

$$\Rightarrow F = (x + y + z)^2(x + y' + z)(x' + y + z)^2(x' + y + z') = M_0 M_2 M_4 M_5$$

$$\Rightarrow F(x, y, z) = \prod(0, 2, 4, 5) \quad \prod : \text{ANDing of terms}$$

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Conversion between Canonical Forms

- The complement of a function = the sum of minterms missing from the original function
 - $F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$
 - $F'(A, B, C) = \Sigma(0, 2, 3) = m_0 + m_2 + m_3$
- From DeMorgan's theorem:
 - $F = (m_0 + m_2 + m_3)' = m_0' m_2' m_3' = M_0 M_2 M_3 = \prod(0, 2, 3)$
 - $m_j' = M_j$ are shown in Table 2-3
- To convert from one canonical form to another:
 - Interchange the symbol Σ and \prod
 - List those numbers missing from the original form

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Example 2-5 by Conversion

- $$F = xy + x'z = x'y'z + x'yz + xyz' + xyz$$

x	y	z	F	
0	0	0	0	
0	0	1	1	→ m_1
0	1	0	0	
0	1	1	1	→ m_3
1	0	0	0	
1	0	1	0	
1	1	0	1	→ m_6
1	1	1	1	→ m_7

→ $\Sigma(1,3,6,7)$

- The missing numbers are 0, 2, 4, 5
 - $$F = \prod(0,2,4,5)$$

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Standard Forms

- The canonical forms are basic forms obtained from the truth table
 - Very seldom to have the least number of literals
- Standard forms : not required to have all variables in each term
 - Sum of products** [ex: $F_1 = y' + xy + x'yz'$]
 - Product of sums** [ex: $F_2 = x(y' + z)(x' + y + z')$]
- Results in a two-level gating structure

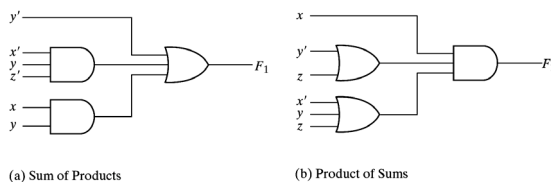


Fig. 2-3 Two-level implementation

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Non-Standard Forms

- Neither in sum of products nor in product of sums
 - $F_3 = AB + C(D+E)$ non-standard form
 - $F_3 = AB + CD + CE$ standard form
- Results in a multi-level gating structure
- In general, two-level implementations are preferred
 - Produce the least amount of delay from inputs to outputs

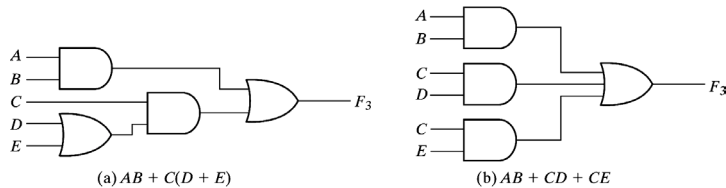


Fig. 2-4 Three- and Two-Level implementation

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Functions of Two Variables

- There are 2^{2^n} functions for n binary variables
 - For 2 variables, there are 16 functions as shown below
- We can assign special operator symbols for each function
 - They can still be expressed by AND, OR, NOT
 - Except the exclusive-OR (\oplus), those new symbol are not commonly used by digital designers

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

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Table 2-8

Boolean functions	Operator symbol	Name	Comments
F ₀ = 0		Null	Binary constant 0
F ₁ = xy	x * y	AND	x and y
F ₂ = xy'	x / y	Inhibition	x, but not y
F ₃ = x		Transfer	x
F ₄ = x'y	y / x	Inhibition	y, but not x
F ₅ = y		Transfer	y
F ₆ = xy' + x'y	x ⊕ y	Exclusive-OR	x or y, but not both
F ₇ = x + y	x + y	OR	x or y
F ₈ = (x + y)'	x ↓ y	NOR	Not-OR
F ₉ = xy + x'y'	(x ⊕ y)'	Equivalence	x equals y
F ₁₀ = y'	y'	Complement	Not y
F ₁₁ = x + y'	x ⊃ y	Implication	If y, then x
F ₁₂ = x'	x'	Complement	Not x
F ₁₃ = x' + y	x ⊃ y	Implication	If x, then y
F ₁₄ = (xy)'	x ↑ y	NAND	Not-AND
F ₁₅ = 1		Identity	Binary constant 1








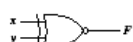
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Digital Logic Gates

- Considerations for constructing other logic gates:
 - The feasibility and economy of producing the gate with physical components
 - The possibility of extending to more than two inputs
 - The basic properties (commutativity, associativity, ...)
 - The ability to implement Boolean functions
- In the 16 functions defined in Table 2-8:
 - Two are equal to a constant and four are repeated twice
 - Two (inhibition and implication) are not commutative or associative
 - The other eight are used as standard gates

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Standard Gates

Name	Graphic symbol	Algebraic function	Truth table	Name	Graphic symbol	Algebraic function	Truth table																														
AND		$F = xy$	<table border="1"> <tr><td>x</td><td>y</td><td>F</td></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	x	y	F	0	0	0	0	1	0	1	0	0	1	1	1	NAND		$F = (xy)'$	<table border="1"> <tr><td>x</td><td>y</td><td>F</td></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	x	y	F	0	0	1	0	1	1	1	0	1	1	1	0
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OR		$F = x + y$	<table border="1"> <tr><td>x</td><td>y</td><td>F</td></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	1	NOR		$F = (x + y)'$	<table border="1"> <tr><td>x</td><td>y</td><td>F</td></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	0
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Inverter		$F = x'$	<table border="1"> <tr><td>x</td><td>F</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </table>	x	F	0	1	1	0	Exclusive-OR (XOR)		$F = xy' + x'y = x \oplus y$	<table border="1"> <tr><td>x</td><td>y</td><td>F</td></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	0									
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Buffer		$F = x$	<table border="1"> <tr><td>x</td><td>F</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> </table>	x	F	0	0	1	1	Exclusive-NOR or equivalence		$F = xy + x'y' = (x \oplus y)'$	<table border="1"> <tr><td>x</td><td>y</td><td>F</td></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	1									
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*Bubble: inversion Triangle: transfer

- NAND and NOR gates are more popular than AND and OR gates because they are more easily constructed with transistor circuits

2-42

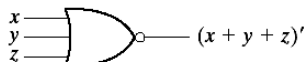
Extension to Multiple Inputs (1/3)

- A gate can be extended to have multiple inputs if the binary operation it represents is commutative and associative
 - The standard gates, except the inverter and buffer, can be extended to have more than two inputs
- The AND and OR operations possess these two properties
 - $X+Y = Y+X$ (commutative)
 - $(X+Y)+Z = X+(Y+Z) = X+Y+Z$ (associative)
 - Can be extended to more than two inputs

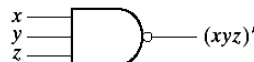
2-43

Extension to Multiple Inputs (2/3)

- The NAND and NOR functions are commutative but not associative
 - $(x \downarrow y) \downarrow z = [(x + y)' + z]' = (x + y)z' = xz' + yz'$
 - $x \downarrow (y \downarrow z) = [x + (y + z)']' = x'(y + z) = x'y + x'z$
- Therefore, the multiple-input NOR (or NAND) gate as a complemented OR (or AND) gate
 - $x \downarrow y \downarrow z = (x+y+z)'$
 - $x \uparrow y \uparrow z = (xyz)'$



(a) 3-input NOR gate



(b) 3-input NAND gate

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Extension to Multiple Inputs (3/3)

- Exclusive-OR (XOR) and equivalence (XNOR) gates are both commutative and associative
 - Can be extended to more than two inputs
- Multiple-input XOR gates are very uncommon
 - Usually constructed with other gates for easier implementation

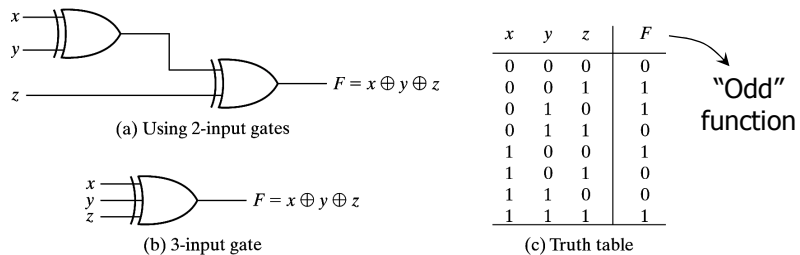


Fig. 2-8 3-input exclusive-OR gate

Logic Polarity

- Positive logic system:
 - Choose the high-level H as logic 1
 - Choose the low-level L as logic 0
- Negative logic system:
 - Choose the high-level H as logic 0
 - Choose the low-level L as logic 1

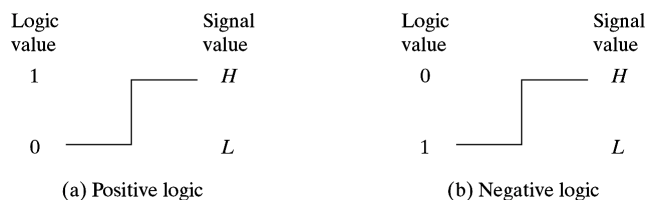


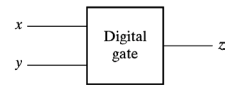
Fig. 2-9 signal assignment and logic polarity

Positive and Negative Logic

- The small triangles in the inputs and output designate a *polarity indicator*
- The same physical gate can operate either as a positive logic AND gate or as a negative logic OR gate
- To convert between them:
 - Interchange 1's and 0's
 - Take the dual function

x	y	F
L	L	L
L	H	L
H	L	L
H	H	H

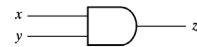
(a) Truth table with H and L



(b) Gate block diagram

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

(c) Truth table for positive logic



(d) Positive logic AND gate

x	y	z
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic



(f) Negative logic OR gate

Fig. 2-10 Demonstration of positive and negative logic

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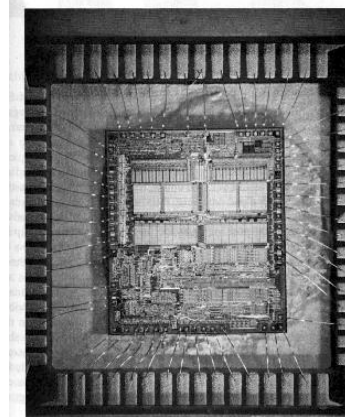
Outline

- Basic Definitions
- Basic Theorems and Properties
- Boolean Functions
- Canonical and Standard Forms
- Digital Logic Gates
- Integrated Circuits

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Integrated Circuits

- An integrated circuit (**IC**) is a silicon semiconductor crystal, called a *chip*, containing the electronic components for constructing digital gates
- The chip is mounted in a ceramic or plastic container
- Connections are welded to external pins from the chip
- No. of pins may range from 14 to several thousands



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Level of Integration

- Small-scale integration (SSI):
 - Contain several independent gates in a package
 - The I/O of the gates are connected directly to external pins
- Medium-scale integration (MSI):
 - Approximate 10 to 1000 gates in a package
 - Usually perform specific elementary operations (adder, ...)
- Large-scale integration (LSI):
 - Contain thousands of gates within a package
 - Include digital systems (processors, memory, ...)
- Very large-scale integration (VLSI):
 - Contain hundred of thousands of gates within a package
- Ultra large-scale integration (ULSI), ...

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Digital Logic Families

- Logic families: classified by the specific circuit technology for implementing the logic
- Many logic families have been used commercially
 - TTL (transistor-transistor logic):
 - Standard logic family
 - ECL (emitter-coupled logic):
 - For high-speed operations
 - MOS (metal-oxide semiconductor):
 - High component density
 - CMOS (complementary MOS):
 - Low power consumption
 - The dominant logic family in VLSI design

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Important Parameters for ICs

- Fan-out
 - No. of standard loads that the output can drive without impairing its normal operation
- Fan-in
 - No. of inputs available in a gate
- Power dissipation
 - The power consumed by the gate
- Propagation delay
 - The average transition delay time for the signal to propagate from input to output
- Noise margin
 - The maximum external noise voltage that does not cause an undesirable change in the circuit output

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Computer-Aided Design (CAD)

- VLSI circuits contain million of transistors
 - Impossible to develop and verify without the assistance of **CAD** tools
- Electronic design automation (**EDA**) covers all phase of the design of integrated circuits
- An important development is the use of a ***hardware description language*** (HDL)
 - Describe the circuits by a formal language
 - Can be used to simulate the system before its construction to check the functionality
 - Translated to real circuits automatically by ***logic synthesis*** tools