Chapter 4

Combinational Logic

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Outline

- $\boxed{\text{Combinational Circuits}}$
- Analysis and Design Procedures
- Binary Adders
- Other Arithmetic Circuits
- Decoders and Encoders
- Multiplexers

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Combinational v.s Sequential Circuits

- Logic circuits may be ***combinational*** or ***sequential***
- Combinational circuits:
  - Consist of ***logic gates*** only
  - Outputs are determined from the present values of inputs
  - Operations can be specified by a set of Boolean functions
- Sequential circuits:
  - Consist of ***logic gates*** and ***storage elements***
  - Outputs are a function of the inputs and the state of the storage elements
    - Depend not only on present inputs, but also on past values
  - Circuit behavior must be specified by a time sequence of inputs and internal states

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Combinational Circuit (1/2)

- A combinational circuit consists of
  - Input variables
  - Logic gates
  - Output variables

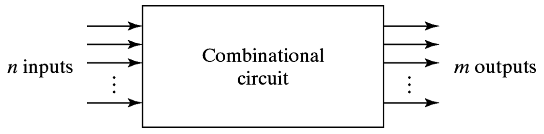

Fig. 4-1 Block Diagram of Combinational Circuit

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Combinational Circuit (2/2)

- Each input and output variable is a binary signal
  - Represent logic 1 and logic 0
- There are 2^n possible binary input combinations for n input variable
- Only one possible output value for each possible input combination
- Can be specified with a truth table
- Can also be described by m Boolean functions, one for each output variable
  - Each output function is expressed in terms of n input variables

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Analysis Procedure

- Analysis: determine the function that the circuit implements
  - Often start with a given logic diagram
- The analysis can be performed by
  - Manually finding Boolean functions
  - Manually finding truth table
  - Using a computer simulation program
- First step: make sure that circuit is combinational
  - Without feedback paths or memory elements
- Second step: obtain the output Boolean functions or the truth table

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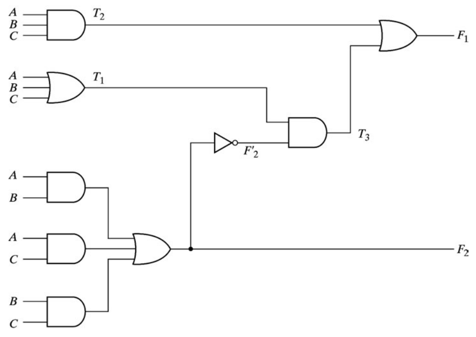
Output Boolean Functions (1/3)

Step 1:

- Label all gate outputs that are a function of input variables
- Determine Boolean functions for each gate output



$F_2 = AB + AC + BC$
$T_1 = A + B + C$
$T_2 = ABC$

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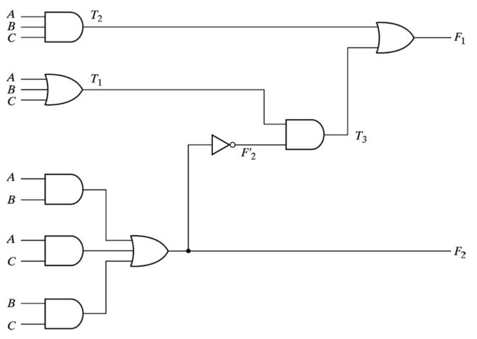
Output Boolean Functions (2/3)

Step 2:

- Label the gates that are a function of input variables and previously labeled gates
- Find the Boolean function for these gates



$T_3 = F'_2\, T_1$
$F_1 = T_3 + T_2$

Output Boolean Functions (3/3)

Step 3:

- Obtain the output Boolean function in term of input variables
  - By repeated substitution of previously defined functions

$$
\begin{aligned}
F_1 = T_3 + T_2 &= F'_2\, T_1 + ABC \\
&= (AB + AC + BC)'\,(A + B + C) + ABC \\
&= (A' + B')(A' + C')(B' + C')\,(A + B + C) + ABC \\
&= (A' + B'C')(AB' + AC' + BC' + B'C) + ABC \\
&= A'BC' + A'B'C + AB'C' + ABC
\end{aligned}
$$

Truth Table

- To obtain the truth table from the logic diagram:
  1. Determine the number of input variables
     For n inputs:
     - 2^n possible combinations
     - List the binary numbers from 0 to 2^n-1 in a table
  2. Label the outputs of selected gates
  3. Obtain the truth table for the outputs of those gates that are a function of the input variables only
  4. Obtain the truth table for those gates that are a function of previously defined variables at step 3
     - Repeatedly until all outputs are determined

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Truth Table for Fig. 4-2

A	B	C	F_2	F'_2	T_1	T_2	T_3	F_1
0	0	0	0	1	0	0	0	0
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	1	1
0	1	1	1	0	1	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	1	0	1	0	0	0
1	1	0	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1

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Design Procedure

- Design procedure:
  - Input: the specification of the problem
  - Output: the logic circuit diagram (or Boolean functions)
- Step 1: determine the required number of inputs and outputs from the specification
- Step 2: derive the truth table that defines the required relationship between inputs and outputs
- Step 3: obtain the simplified Boolean function for each output as a function of the input variables
- Step 4: draw the logic diagram and verify the correctness of the design

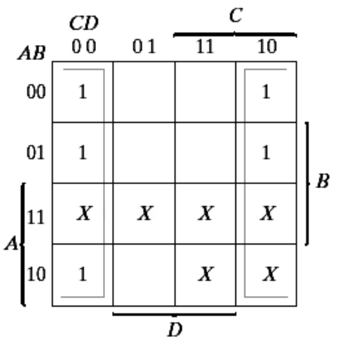
Code Conversion Example

- Convert from BCD code to Excess-3 code
- The 6 input combinations not listed are don't cares
- These values have no meaning in BCD
- We can arbitrary assign them to 1 or 0

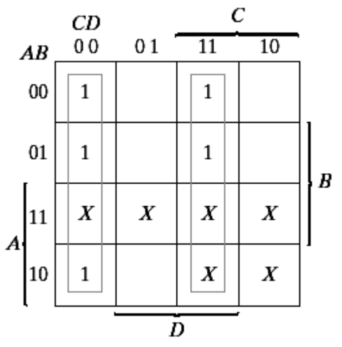
Input BCD				Output Excess-3 Code			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Maps for Code Converter (1/2)

- The six don’t care minterms (10~15) are marked with X

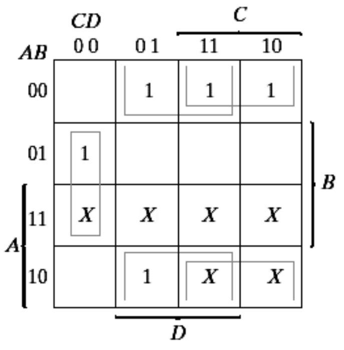


$z = D'$

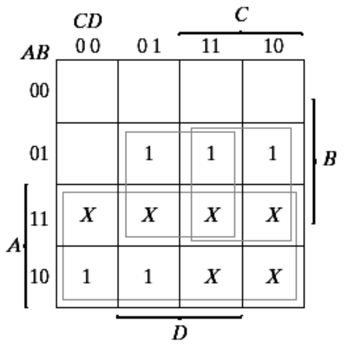


$y = CD + C'D'$

Maps for Code Converter (2/2)



$X = B'C + B'D + BC'D'$



$w = A + BC + BD$

Logic Diagram for the Converter

- There are various possibilities for a logic diagram that implements a circuit
- A two-level logic diagram may be obtained directly from the Boolean expressions derived by the maps
- The expressions may be manipulated algebraically to use common gates for two or more outputs
  - Reduce the number of gates used

z = D'
y = CD + C' D' = CD + (C + D) '
x = B'C + B'D + BC' D' = B' (C + D) + BC' D'
  = B' (C + D) + B(C + D)'
w = A + BC + BD = A + B(C + D)

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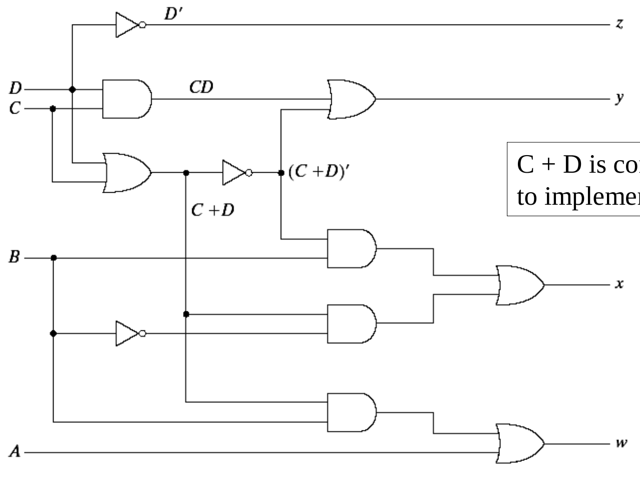
Logic Diagram for the Converter



C + D is commonly used
to implement the three outputs

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Adder

- The most basic arithmetic operation is the addition of two binary digits
  - When both augend and addend bits are equal to 1, the binary sum consists of two digits (1 + 1 = 10)
  - The higher significant bit of this result is called a ***carry***
- A combination circuit that performs the addition of two bits is ***half adder***
- A adder performs the addition of 2 significant bits and a previous carry is called a ***full adder***

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Half Adder

- Half adder
  - Inputs: x and y
  - Outputs: S (for sum) and C (for carry)

x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

S = $x'y + xy'$
C = xy

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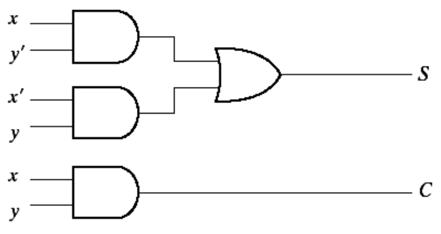
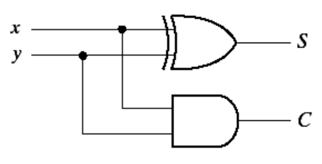
Implementation of a Half Adder



(a) $S = xy' + x'y$
$C = xy$



(b) $S = x \oplus y$
$C = xy$

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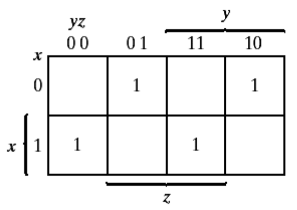
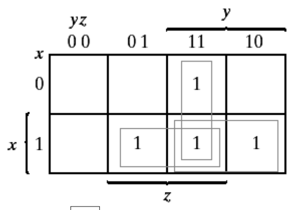
Full Adder

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$S = x'y'z + x'yz' + xy'z' + xyz$



$\boxed{C} = xy + xz + yz$
$= xy + xy'z + x'yz$

x, y: the two significant bits to be added
z: the carry from the previous position

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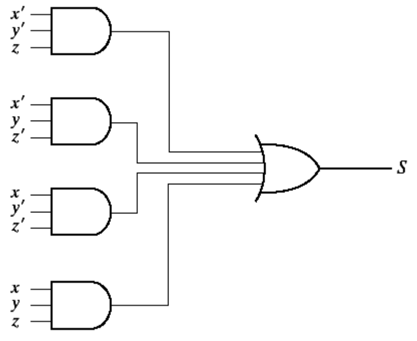
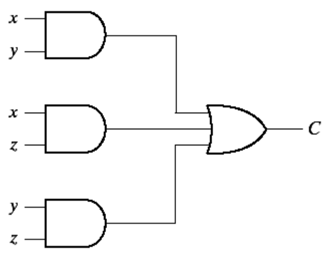
Implementation of a Full Adder





$C = xy + xz + yz$

$S = x'y'z + x'yz' + xy'z' + xyz$

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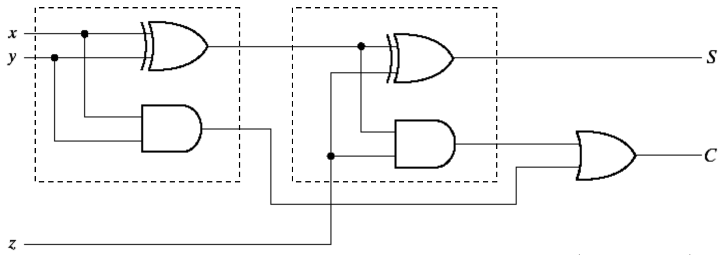
Implementation of a Full Adder

- A full adder can be implemented with two half adders and an OR gate



$$S = z \oplus (x \oplus y)$$
$$= z'(xy' + x'y) + z(xy' + x'y)'$$
$$= z'(xy' + x'y) + z(xy + x'y')$$
$$= xy'z' + x'yz' + xyz + x'y'z$$

$$C = z(xy' + x'y) + xy$$
$$= xy'z + x'yz + xy$$
$$= xy'z + x'yz + xyz + xyz'$$
$$= xz + yz + xy$$

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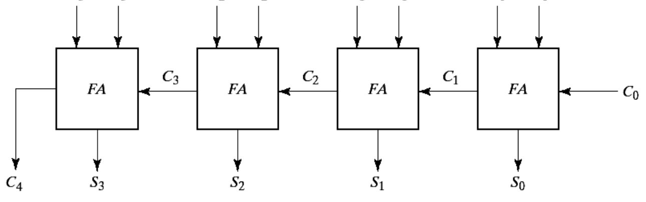
Binary Adder

- A binary adder produces the arithmetic sum of two binary numbers
- Can be constructed with full adders connected in cascade
  - The output carry from each full adder is connected to input carry of the next full adder in the chain
  - n-bit binary ***ripple carry adder*** is connected by n FAs



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4-bit Adder Example

- Consider two binary number A = 1011 and B = 0011

Subscript i :	**3**	**2**	**1**	**0**	
Input carry	0	1	1	0	C_i
Augend	1	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output carry	0	0	1	1	C_{i+1}

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Carry Propagation

- As in a combinational circuit, the signal must propagate through the gates before the correct output sum is available in the output terminals
- The total propagation time is equal to the propagation delay of a typical gate times the number of levels in the circuit
- The longest propagation delay in a adder is the time that carry propagate through the full adders
- Each bit of the sum output depends on the value of the input carry
  - The value of S_i will be in final value only after the input carry C_i has been propagated

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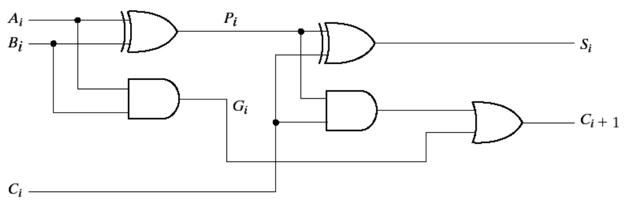
Full Adder with P and G

- The full adder can be redrawn with two internal signals P (propagation) and G (generation)
- The signal from input carry C_i to output carry C_{i+1} propagates through an AND and a OR gate (2 gate levels)
  - For n-bit adder, there are 2n gate levels for the carry to propagate from input to output



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Carry Propagation

- The ***carry propagation time*** is a limiting factor on the speed with which two numbers are added
- All other arithmetic operations are implemented by successive additions
  - The time consumed during the addition is very critical
- To reduce the carry propagation delay
  - Employ faster gates with reduced delays
  - Increase the equipment complexity
- Several techniques for reducing the carry propagation time in a parallel adder
  - The most widely used technique employs the principle of ***carry lookahead***

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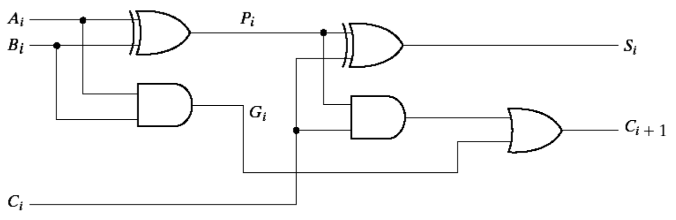
Carry Propagation & Generation



carry propagate : $P_i = A_i \oplus B_i$ $\quad S_i = P_i \oplus C_i$

carry generate : $G_i = A_iB_i$ $\quad C_{i+1} = G_i + P_iC_i$

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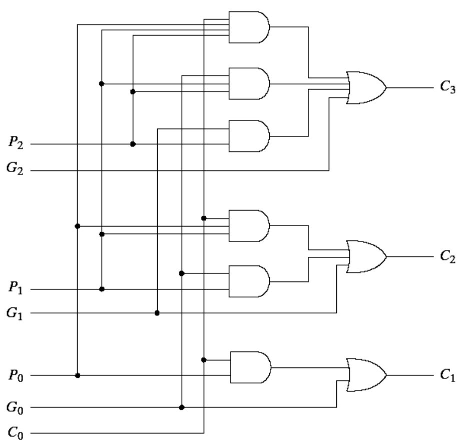
Carry Lookahead Generator



C_0 = input carry

$C_1 = G_0 + P_0C_0$

$C_2 = G_1 + P_1C_1$

$= G_1 + P_1(G_0 + P_0C_0)$

$= G_1 + P_1G_0 + P_1P_0C_0$

$C_3 = G_2 + P_2C_2$

$= G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0$

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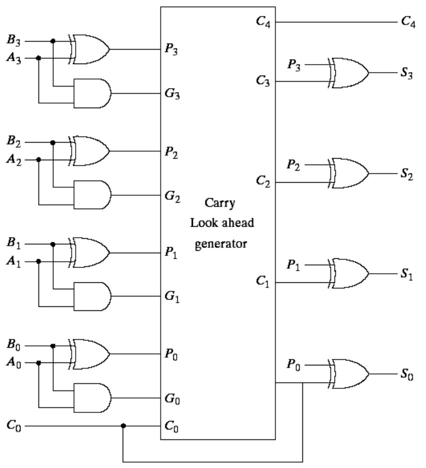
Carry Lookahead Adder



- All output carries are generated after a delay through two levels of gates
- Output S1 to S3 can have equal propagation delay times

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Binary Subtractor

- A – B can be done by taking the 2's complement of B and adding it to A ---> A – B = A + (-B)
  - 2'complement can be obtained by taking the 1'complement and adding on to the least significant pair of bits
  - A - B = A + (B′ + 1)
- The circuit for subtraction A – B consists of an adder with inverter placed between each data input B and the corresponding input of the full adder
- The input carry C_0 must be equal to 1

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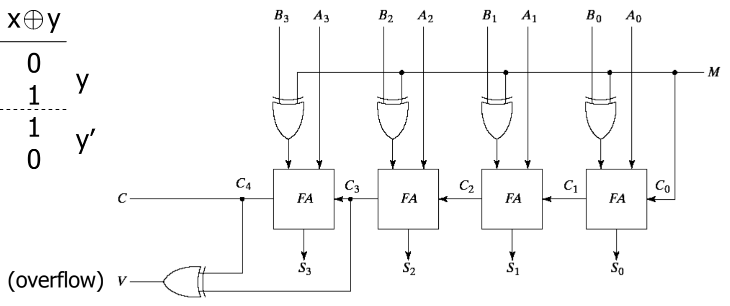
4-bit Adder-Subtractor

- M=0 (Adder)
  - Input of FA is A and B (B ⊕ 0 = B), and C_0 is 0
- M=1 (Subtractor)
  - Input of FA is A and B′ (B ⊕ 1 = B′), and C_0 is 1

x	y	x⊕y	
0	0	0	y
0	1	1	
1	0	1	y′
1	1	0	


B_3 A_3 B_2 A_2 B_1 A_1 B_0 A_0
M
C_4 C_3 C_2 C_1 C_0
C
FA FA FA FA
S_3 S_2 S_1 S_0
(overflow) V

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Overflow

- An overflow occurs when two number of *n* digits each are added and the sum occupies *n+1* digits
- When two unsigned numbers are added, an overflow is detected from the end carry out of the most significant position
- When two signed numbers are added, the sign bit is treated as part of the number and the end carry does not indicate an overflow
  - Extra overflow detection circuits are required
- An overflow can only occur when two numbers added are ***both positive*** or ***both negative***

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Overflow Example

The two carry bits are different !!

```
Carries :  0   1                        Carries :  1   0
   +70         0   1000110                 -70         1   0111010
   +80         0   1010000                 -80         1   0110000
--------      ----------------         --------       ----------------
 +150          1   0010110 (-106)        -150          0   1101010 (+106)
(010010110)                            (101101010)
```

Overflow

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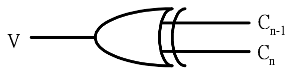
Overflow Detection

- An overflow condition can be detected by observing the carry into the sign bit position and the carry out of the sign bit position
  - If these two carries are not equal, and overflow has occurred
  - If the output V is equal to 1, an overflow is detected



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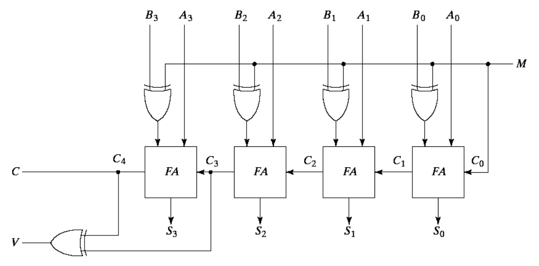
Adder-Subtractor Circuit

- Unsigned
  - C bit detects a ***carry*** after addition or a ***borrow*** after subtraction
- Signed
  - V bit detects an overflow

    0: no overflow; 1: overflow



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Decimal Adder

- A decimal adder requires a minimum of 9 inputs and 5 outputs
  - 1 digit requires 4-bit
  - Input: 2 digits + 1-bit carry
  - Output: 1 digit + 1-bit carry
- BCD adder
  - Perform the addition of two decimal digits in BCD, together with an input carry from a previous stage
  - The output sum cannot be greater than 19 (9+9+1)

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Derivation of BCD Adder

Binary Sum					BCD Sum					Decimal
K	Z_8	Z_4	Z_2	Z_1	C	S_8	S_4	S_2	S_1	
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	1
		…………					…………			……
0	1	0	1	0	1	0	0	0	0	10
0	1	0	1	1	1	0	0	0	1	11
0	1	1	0	0	1	0	0	1	0	12
0	1	1	0	1	1	0	0	1	1	13
0	1	1	1	0	1	0	1	0	0	14
0	1	1	1	1	1	0	1	0	1	15
1	0	0	0	0	1	0	1	1	0	16
1	0	0	0	1	1	0	1	1	1	17
1	0	0	1	0	1	1	0	0	0	18
1	0	0	1	1	1	1	0	0	1	19

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BCD Adder

- When the binary sum is equal to or less than 1001_b
  - BCD Sum = Binary Sum
  - C = 0
- When the binary sum is greater than 1001_b
  - BCD Sum = Binary Sum + 0110_b
  - C = 1

$Z_2Z_1 \backslash Z_8Z_4$	00	01	11	10
00			1	
01			1	
11			1	1
10			1	1

$C = K + Z_8Z_4 + Z_8Z_2$

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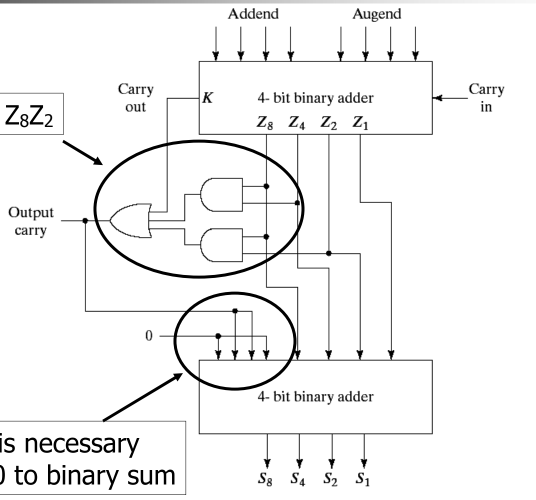
Block Diagram of a BCD Adder

$C = K + Z_8Z_4 + Z_8Z_2$



If C =1 , it is necessary to add 0110 to binary sum

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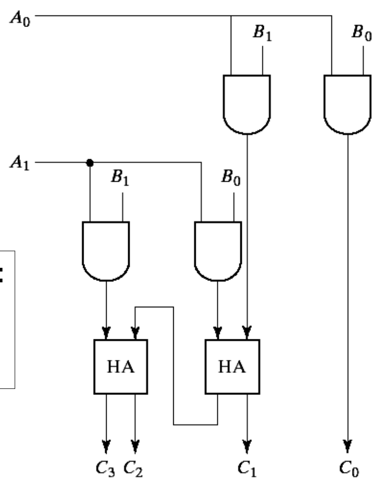
Binary Multiplier

$$\begin{array}{cccc}
 & & B_1 & B_0 \\
 & & A_1 & A_0 \\
\hline
 & & A_0B_1 & A_0B_0 \\
 & A_1B_1 & A_1B_0 & \\
\hline
C_3 & C_2 & C_1 & C_0
\end{array}$$



For a J*K bits multiplier , we need:
(J * K) AND gates
(J – 1) K-bit adders
to produce a product of J+K bits

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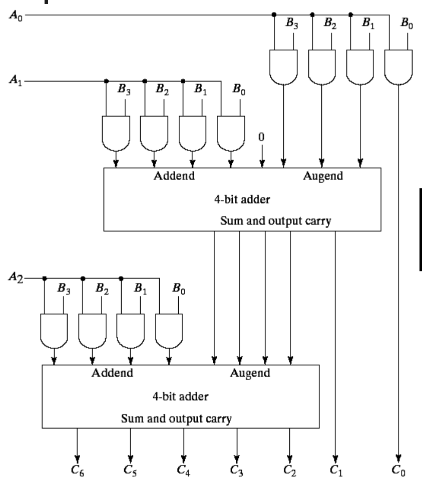
4-Bit By 3-Bit Binary Multiplier



$$\begin{array}{ccccccc}
 & & & B_3 & B_2 & B_1 & B_0 \\
 & & & & A_2 & A_1 & A_0 \\
\hline
 & & 0 & B_3A_0 & B_2A_0 & B_1A_0 & B_0A_0 \\
 & \text{carryout} & B_3A_1 & B_2A_1 & B_1A_1 & B_0A_1 & \\
\text{carryout} & B_3A_2 & B_2A_2 & B_1A_2 & B_0A_2 & & \\
\hline
C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0
\end{array}$$

We need 12 AND gates and two 4-bit adders to produce a product of 7 bits

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Magnitude Comparator

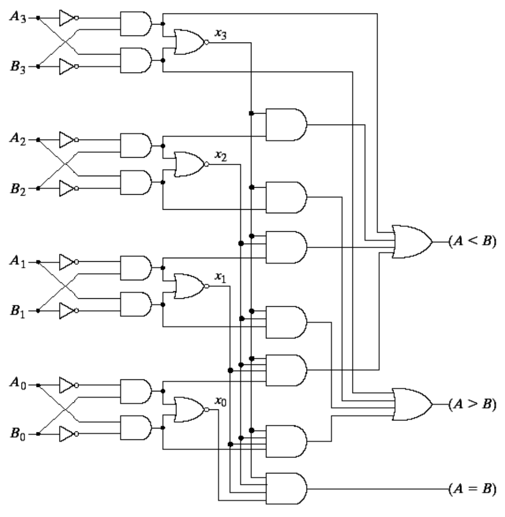
- Equal (A = B)
  - $A_3=B_3$ and $A_2=B_2$ and $A_1=B_1$ and $A_0=B_0$

    $\boxed{X_i = A_iB_i + A_i'B_i' \text{ for } i = 0,1,2,3}$ ← Xi = 1 means Ai and Bi are equal !!
  - $(A=B) = X_3X_2X_1X_0$
- Greater (A > B) or Less (A < B)
  - Comparison start from the MSB
  - If the two digits are equal, compare the next lower digits
  - Continues until a pair of unequal digits is reached
    - A is 1 and B is 0 => A > B
    - A is 0 and B is 1 => A < B

$$(A > B) = A_3B_3' + X_3A_2B_2' + X_3X_2A_1B_1' + X_3X_2X_1A_0B_0'$$
$$(A < B) = A_3'B_3 + X_3A_2'B_2 + X_3X_2A_1'B_1 + X_3X_2X_1A_0'B_0$$

4-47

4-Bit Magnitude Comparator



4-48

Outline

4-49

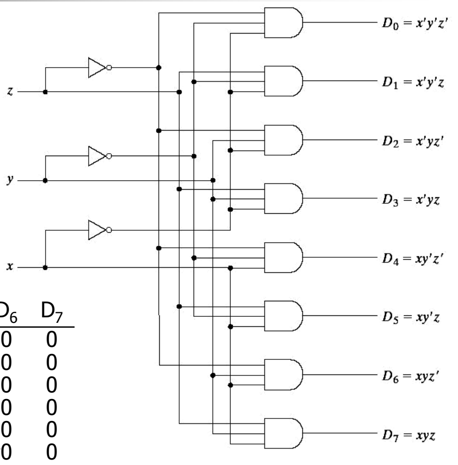
Decoder

- A circuit that coverts binary information from n input lines to a maximum of 2^n unique output lines
  - May have fewer than 2^n outputs
- A n-to-m-line decoder ($m \leq 2^n$):
  - Generate the m minterns of n input variables
- For each possible input combination, there is only one output that is equal to 1
  - The output whose value is equal to 1 represents the minterm equivalent of the binary number presently available in the input lines

4-50

3-to-8-Line Decoder

- The 3 inputs are decoded into 8 outputs
- Each represent one of the minterms of the inputs variables

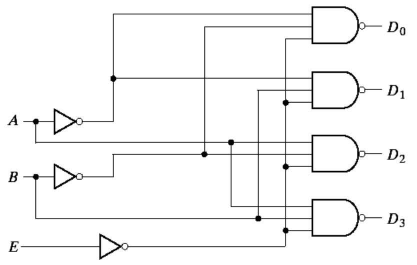


Inputs			Outputs							
x	y	z	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

4-51

2-to-4-Line Decoder with Enable

- Some decoders are constructed with NAND gates
  - Generate minterms in their complement form
- An ***enable*** input can be added to control the operation
  - E=1: disabled
  - None of the outputs are equal to 0

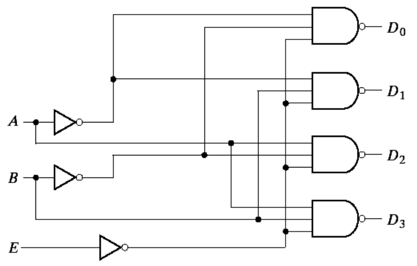


E	A	B	D_0	D_1	D_2	D_3
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

4-52

Demultiplexer

- A circuit that receives information from a single line and directs it to one of 2^n possible output lines
- A decoder with enable input can function as a demultiplexer
  - Often referred to as a ***decoder/demultiplexer***



data input
selection

E	A	B	D_0	D_1	D_2	D_3
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

Construct Larger Decoders

- Decoders with enable inputs can be connected together to form a larger decoder
- The enable input is used as the most significant bit of the selection signal
  - w=0: the top decoder is enabled
  - w=1: the bottom one is enabled
- In general, enable inputs are a convenient feature for standard components to expand their numbers of inputs and outputs

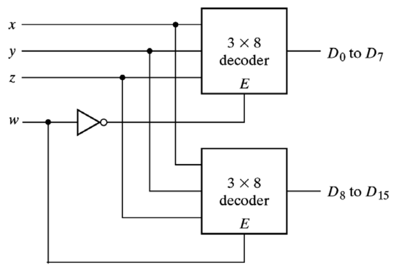
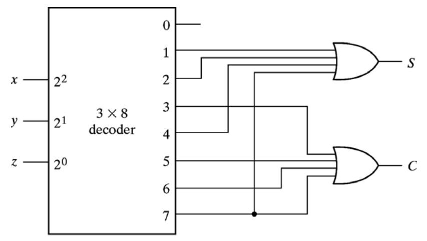


Fig. 4-20 4×16 Decoder Constructed with Two 3×8 Decoders

Combinational Logic Implementation

- A decoder provides the 2^n minterms of n input variables
  - Can be used to form any combinational circuits with extra OR gates (sum of minterms)
- A function having a list of k minterms can be expressed in its complemented form F' with $2^n - k$ minterms
  - If $k > 2^n/2$, F' will have fewer minterms (fewer OR gates)
  - NOR gates are used instead for implementing F'



$S(x,y,z) = \Sigma(1,2,4,7)$
$C(x,y,z) = \Sigma(3,5,6,7)$

4-55

Encoder

- A circuit that performs the inverse operation of a decoder
  - Have 2^n (or fewer) input lines and n output lines
  - The output lines generate the binary code of the input positions
- Only one input can be active at any given time
- An extra output may be required to distinguish the cases that $D_0 = 1$ and all inputs are 0

Inputs								Outputs		
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	x	y	z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

$z = D_1 + D_3 + D_5 + D_7$
$y = D_2 + D_3 + D_6 + D_7$
$x = D_4 + D_5 + D_6 + D_7$

4-56

Priority Encoder

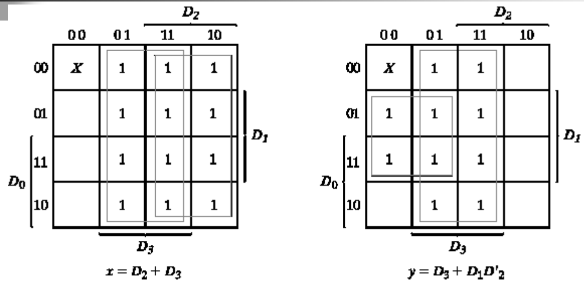
- An encoder circuit that includes the priority function
- If two or more inputs are equal to 1 at the same time, the input having the highest priority will take precedence
- In the following truth table:
  - $D_3 > D_2 > D_1 > D_0$
  - The X's in output columns represent don't-care conditions
  - The X's in input columns are useful for representing a truth table in condensed form

Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

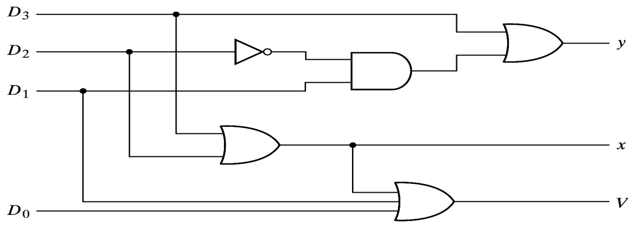
V = 0 :
no valid inputs

4-57

Implement a Priority Encoder



$x = D_2 + D_3$
$y = D_3 + D_1D'_2$
$V = D_0 + D_1 + D_2 + D_3$



4-58

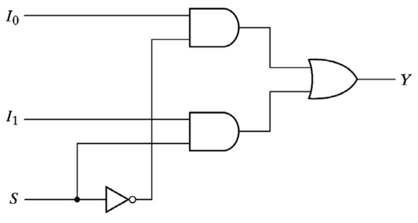
Outline

- Combinational Circuits
- Analysis and Design Procedures
- Binary Adders
- Other Arithmetic Circuits
- Decoders and Encoders
- Multiplexers

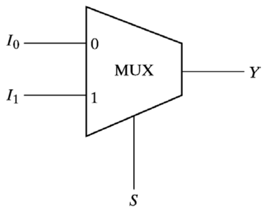
4-59

Multiplexer

- A circuit that selects binary information from one of many input lines and directs it to a single output lines
  - Have 2^n input lines and n selection lines
  - Act like an electronic switch (also called a ***data selector***)
- For the following 2-to-1-line multiplexer:
  - S=0 → Y = I_0 ; S=1 → Y = I_1



(a) Logic diagram

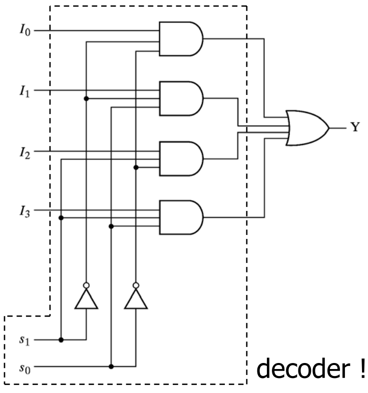


(b) Block diagram

4-60

4-to-1-Line Multiplexer

- The combinations of S0 and S1 control each AND gates
- Part of the multiplexer resembles a decoder
- To construct a multiplexer:
  - Start with an *n*-to-2^n decoder
  - Add 2^n input lines, one to each AND gate
  - The outputs of the AND gates are applied to a single OR gate



decoder !!

(a) Logic diagram

s_1	s_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

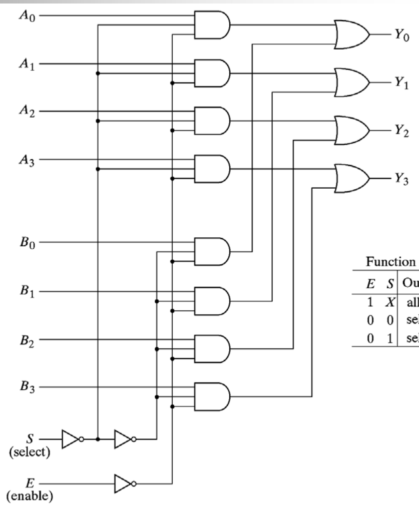
(b) Function table

Quadruple 2-to-1-Line Multiplexer

- Multiplexers can be combined with ***common selection inputs*** to provide multiple-bit selection logic
- Quadruple 2-to-1-line multiplexer:
  - Four 2-to-1-line multiplexers
  - Each capable of selecting one bit of the 2 4-bit inputs
  - E: enable input

    E=1: disable the circuit

    (all outputs are 0)



Function table

E	S	Output Y
1	X	all 0's
0	0	select A
0	1	select B

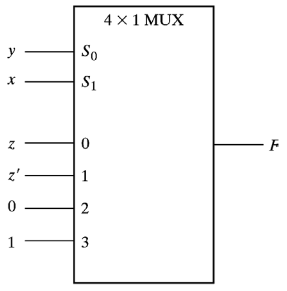
Boolean Function Implementation

- A multiplexer is essentially a decoder with an external OR gate
  - Can be used to implement Boolean functions without extra logic
- To implement a Boolean function of *n* variables:
  - Use a multiplexer with *n* - 1 selection inputs
  - The first *n* – 1 variables are connected to the selection inputs
  - The remaining variable is used for the data inputs

F(x,y,z) = Σ(1,2,6,7)

x	*y*	*z*	*F*	
0	0	0	0	*F* = *z*
0	0	1	1	
0	1	0	1	*F* = *z*′
0	1	1	0	
1	0	0	0	*F* = 0
1	0	1	0	
1	1	0	1	*F* = 1
1	1	1	1	

(a) Truth table



(b) Multiplexer implementation

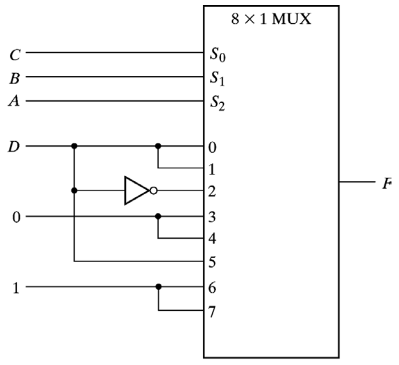
4-63

Implementing a 4-Input Function

F(A,B,C,D) = Σ(1,3,4,11,12,13,14,15)

logic 0: connected to ground
logic 1: connected to Vdd (5V)

A	*B*	*C*	*D*	*F*	
0	0	0	0	0	*F* = *D*
0	0	0	1	1	
0	0	1	0	0	*F* = *D*
0	0	1	1	1	
0	1	0	0	1	*F* = *D*′
0	1	0	1	0	
0	1	1	0	0	*F* = 0
0	1	1	1	0	
1	0	0	0	0	*F* = 0
1	0	0	1	0	
1	0	1	0	0	*F* = *D*
1	0	1	1	1	
1	1	0	0	1	*F* = 1
1	1	0	1	1	
1	1	1	0	1	*F* = 1
1	1	1	1	1	



can be 0, 1, the variable, or its complement

4-64

Three-State Gate

- A circuit that exhibits three states
  - logic 1, logic 0, and ***high-impedance (z)***
- The high-impedance state acts like an open circuit (disconnected)
- The most commonly used three-state gate is the buffer gate
  - C=0 → disabled (high-impedance) ; C=1 → enabled (pass)
  - Can be used at the output of a function without altering the internal implementation

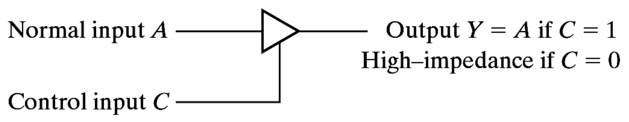
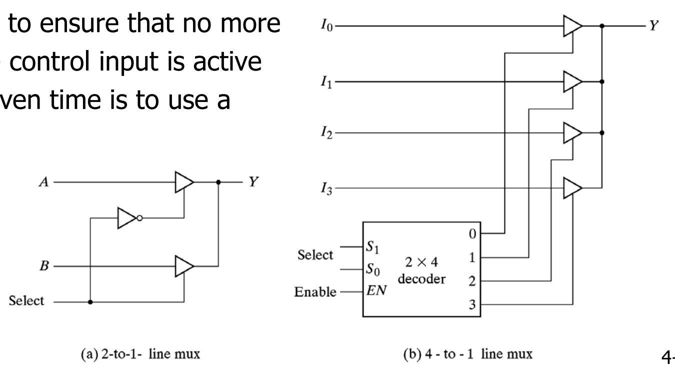


Fig. 4-29 Graphic Symbol for a Three-State Buffer

4-65

Implementation with 3-State Gates

- A large number of three-state gate outputs can be connected with wires to form a common line (bus) without logic conflicts
  - Very convenient for implementing some circuits (ex: multiplexer)
  - Only one buffer can be in the active state at any given time
- One way to ensure that no more than one control input is active at any given time is to use a decoder



(a) 2-to-1- line mux (b) 4 - to - 1 line mux

4-66